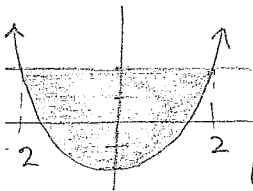


Problem Set 25.
(7.2) Part 2

1. $y = x^2 - 2, y = 2.$



$$x^2 - 2 = 2$$

$$x^2 = 4$$

$$x = \pm 2$$

Use symmetry

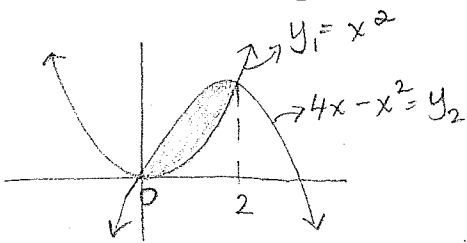
$$A = 2 \int_0^2 (2 - (x^2 - 2)) dx = 2 \int_0^2 (2 - x^2 + 2) dx$$

$$= 2 \int_0^2 (4 - x^2) dx = 2 \left[4x - \frac{1}{3}x^3 \right]_0^2$$

$$= 2 \left[4(2) - \frac{1}{3}(2)^3 \right] - 0$$

$$= 2 \left(8 - \frac{8}{3} \right) = \left(\frac{32}{3} \right)$$

2. $y = x^2; y = 4x - x^2$



$$4x - x^2 = x^2$$

$$4x - 2x^2 = 0$$

$$2x(2 - x) = 0$$

$$\downarrow x = 2$$

$$A = \int_0^2 (4x - x^2 - (x^2)) dx$$

$$= \int_0^2 (4x - x^2 - x^2) dx$$

$$= \int_0^2 (4x - 2x^2) dx$$

3 contd

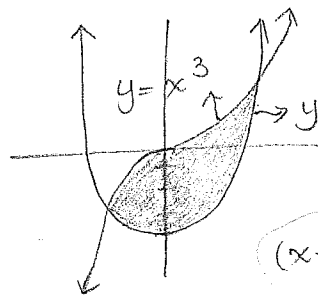
$$= \left[\frac{1}{2}(4)x^2 - \frac{1}{3}(2)x^3 \right]_0^2$$

$$= \left[2x^2 - \frac{2}{3}x^3 \right]_0^2 =$$

$$= \left[2(2)^2 - \frac{2}{3}(2)^3 \right] - 0$$

$$= \frac{8}{3} - \frac{16}{3} = \left(\frac{8}{3} \right)$$

3. $y = x^3; y = 3x^2 - 4$



$$3x^2 - 4 = x^3$$

$$-x^3 + 3x^2 - 4 = 0$$

Graph - find zeros.

$$x = -1, \Rightarrow (x+1)$$

$$(x+1) \overline{) -x^3 + 3x^2 + 0x - 4}$$

$$-x^3 + 4x - 4$$

$$x^2 - 4x + 4$$

$$(x-2)(x-2)$$

$$\downarrow x = 2, x = 2.$$

$$+4x^2 + 0x$$

$$+4x^2 + 4x$$

$$-4x - 4$$

$$-4x - 4$$

$$. 0$$

factors $x = 2, 2, -1.$

$$A = \int_{-1}^2 (x^3 - (3x^2 - 4)) dx = \int_{-1}^2 (x^3 - 3x^2 + 4) dx$$

$$= \left[\frac{1}{4}x^4 - \frac{1}{3}(3)x^3 + 4x \right]_{-1}^2 = \left[\frac{1}{4}x^4 - x^3 + 4x \right]_{-1}^2$$

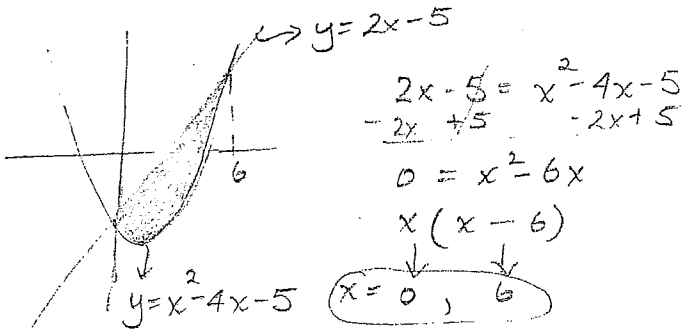
$$= \left[\frac{1}{4}(2)^4 - (2)^3 + 4(2) \right] - \left[\frac{1}{4}(-1)^4 - (-1)^3 + 4(-1) \right]$$

$$= (4 - 8 + 8) - \left(\frac{1}{4} + 1 - 4 \right)$$

$$4 - \left(-2\frac{3}{4} \right)$$

$$4 + 2\frac{3}{4} = \left(6\frac{3}{4} \right)$$

4 $y = x^2 - 4x - 5, y = 2x - 5$



$$2x - 5 = x^2 - 4x - 5$$

$$-2x + 5 = -2x + 5$$

$$0 = x^2 - 6x$$

$$x(x - 6)$$

$x = 0, 6$

$$A = \int_0^6 (2x - 5 - (x^2 - 4x - 5)) dx$$

$$= \int_0^6 (2x - 5 - x^2 + 4x + 5) dx$$

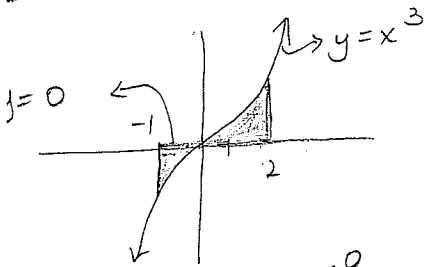
$$= \int_0^6 (-x^2 + 6x) dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{1}{2}(6)x^2 \right]_0^6$$

$$= \left(-\frac{1}{3}(6)^3 + 3(6)^2 \right) - 0$$

$$= -72 + 108 = 36$$

5 $y = x^3, y = 0. [-1, 2]$



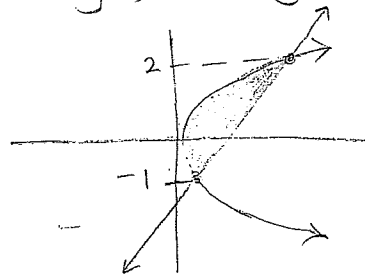
$$A = \int_0^2 x^3 dx + \int_{-1}^0 -x^3 dx$$

$$= \left[\frac{1}{4}x^4 \right]_0^2 - \left[\frac{1}{4}x^4 \right]_{-1}^0$$

$$= \left[\frac{1}{4}(2)^4 - 0 \right] - \left[0 - \frac{1}{4}(-1)^4 \right]$$

$$4 - \left(-\frac{1}{4} \right) = 4\frac{1}{4} \text{ or } \frac{17}{4}$$

6 $x = y^2, x = y + 2$



$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y - 2)(y + 1)$$

$y = \frac{1}{2}, -1$

$$A = \int_{-1}^2 (y + 2 - y^2) dy = \left[\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \right]_{-1}^2$$

$$= \left[\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3 \right] - \left[\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3 \right]$$

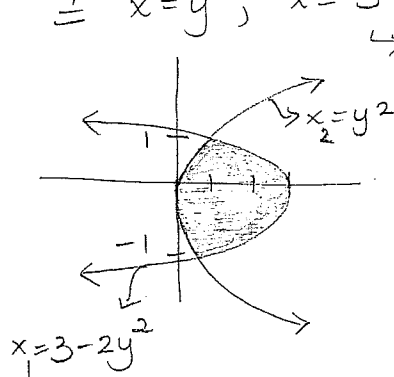
$$= \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} + 1 + \frac{1}{3} \right)$$

$$= \frac{10}{3} - \frac{11}{6} = \frac{9}{6}$$

check that?

$$\frac{10}{3} + \frac{1}{6} = \frac{9}{2} \text{ or } 4\frac{1}{2}$$

7 $x = y^2, x = 3 - 2y^2$



$$3 - 2y^2 - y^2 = 0$$

$$3 - 3y^2$$

$$3(1 - y^2)$$

$$= 3(1 + y)(1 - y)$$

$y = \pm 1$

$$A = \int_{-1}^1 (3 - 2y^2 - y^2) dy = \int_{-1}^1 (3 - 3y^2) dy$$

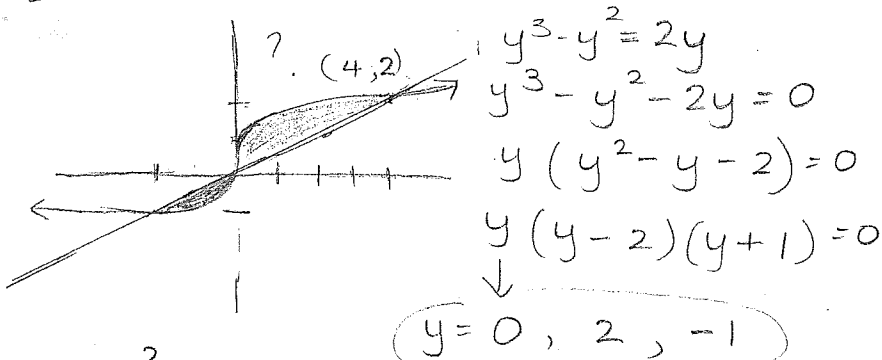
$$\left[3y - \frac{1}{3}(3)y^3 \right]_{-1}^1 = \left[3y - y^3 \right]_{-1}^1$$

$$(3(1) - (1)^3) - (3(-1) - (-1)^3)$$

$$= 2 - (-2)$$

$$= 4$$

8 $x = y^3 - y^2$, $x = 2y$



$$A = \int_0^2 (2y - y^3 + y^2) dy + \int_{-1}^0 (y^3 - y^2 - 2y) dy$$

$$= \left(\frac{1}{2}(2)y^2 - \frac{1}{4}y^4 + \frac{1}{3}y^3 \right) \Big|_0^2 + \left(\frac{1}{4}y^4 - \frac{1}{3}y^3 - \frac{1}{2}(2)y^2 \right) \Big|_{-1}^0$$

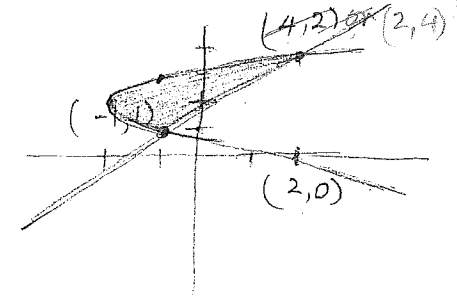
$$= \left[y^2 - \frac{1}{4}y^4 + \frac{1}{3}y^3 \right]_0^2 + \left[\frac{1}{4}y^4 - \frac{1}{3}y^3 - y^2 \right]_{-1}^0$$

$$= \left[(2^2 - \frac{1}{4}(2)^4 + \frac{1}{3}(2)^3) - 0 \right] + \left[0 - (\frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - (-1)^2) \right]$$

$$\frac{8}{3} + \frac{5}{12} = \boxed{\frac{37}{12} \text{ or } 3\frac{1}{12}}$$

9 contd $A = \int_1^4 (y-2 - (y^2 - 4y + 2)) dy$

9 $x = y^2 - 4y + 2$, $x = y - 2$



$$= \int_1^4 (y^2 - 5y + 4) dy$$

$$= \left[\frac{1}{3}y^3 - \frac{5}{2}y^2 + 4y \right]_1^4$$

$$= \left[\frac{1}{3}(4^3) - \frac{5}{2}(4^2) + 4(4) \right] - \left[\frac{1}{3}(1)^3 - \frac{5}{2}(1)^2 + 4(1) \right]$$

$$\left(\frac{64}{3} - 40 + 16 \right) - \left(\frac{1}{3} - \frac{5}{2} + 4 \right)$$

$$\left(-\frac{8}{3} \right) - \left(\frac{11}{6} \right)$$

$$= \boxed{4\frac{1}{2}}$$

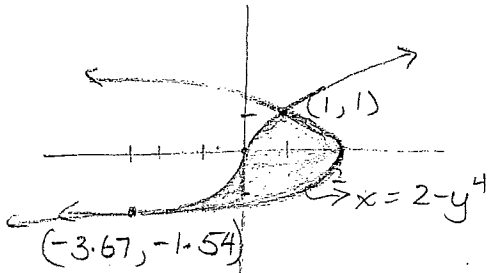
$$y^2 - 4y + 2 = y - 2$$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1)$$

$$y = \boxed{4, 1}$$

10 $x = y^3, x = 2 - y^4$



$y = 1, -1.54$ (Find intersections on calc)

$$A = \int_{-1.54}^1 (2 - y^4 - y^3) dy$$

$$= \left[2y - \frac{1}{5}y^5 - \frac{1}{4}y^4 \right]_{-1.54}^1$$

$$= \left[2(1) - \frac{1}{5}(1)^5 - \frac{1}{4}(1)^4 \right] - \left[2(-1.54) - \frac{1}{5}(-1.54) - \frac{1}{4}(-1.54)^4 \right]$$

$$\left(2 - \frac{1}{5} - \frac{1}{4} \right) - (-3.08 + 1.732 + 1.406)$$

$$= \boxed{1.49}$$