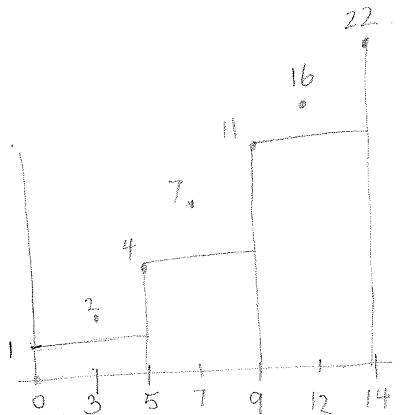
FRQ K1



CALCULUS AB

Part A

Time—25 minutes

Number of problems—1

A GRAPHING CALCULATOR IS NOT REQUIRED FOR THESE PROBLEMS.

t (days)	0	3	5	7	9	12	14
$h(t)$ (# of butterflies)	1	2	4	7	11	16	22

1. The number of butterflies spotted in Amelie's garden each afternoon is modeled by a differentiable and increasing function h for $0 \le t \le 14$, where t is measured in days. Selected values of $h(t)$ are given in the table above.

(a) Use the data in the table to find an approximation for $h'(8)$. Using correct units, interpret the meaning of this value in the context of the problem.

Solution:
$h'(8) \approx \frac{11-7}{9-7} = 2$
On day 8, the number of butterflies spotted in Amelie's garden is increasing at a rate of appproximately 2 butterflies per day.

Score:
1: approximation
1: interpretation with correct units

(b) Explain why there must be at least one time t, for $0 < t < 14$, such that $h'(t) = 1.5$.

Solution:
Since $h(t)$ is continuous and differentiable on $[0, 14]$, the Mean Value Theorem guarantees a value $0 < c < 14$ such that $h'(c) = \frac{h(14)-h(0)}{14-0} = \frac{21}{14} = 1.5$.

Score:
1: $h(t)$ continuous on $[0, 14]$ and differentiable on $(0, 14)$
Note: accept differentiable on $[0, 14]$ since it is given
1: average rate of change on $[0, 14] = 1.5$
1: conclusion

(c) Approximate the value of $\int_0^{14} h(t)\,dt$ using a left Riemann sum on the subintervals $[0,5]$, $[5,9]$, and $[9,14]$. Is this approximation greater than or less than $\int_0^{14} h(t)\,dt$? Give a reason for your answer.

Solution:

$\int_0^{14} h(t)\,dt \approx 5(1) + 4(4) + 5(11) = 76$

This approximation is less than $\int_0^{14} h(t)\,dt$ since $h(t)$ is increasing and we are using a left Riemann sum.

Score:

1: left Riemann sum

1: approximation

1: underestimate with reason

(d) The number of bees spotted in Amelie's garden during the same 14-day period is modeled by the function $g(t) = 7e^{\frac{1}{7}(x-1)} + \cos(2x - 16)$ for $0 \le t \le 14$, where t is measured in days. At time $t = 8$, is the number of bees increasing at an increasing rate, increasing at a decreasing rate, or neither? Based on your answer, what can you conclude about the graph of $g(t)$ at $t = 8$? Justify your answer.

Solution:

$g'(t) = e^{\frac{1}{7}(t-1)} - 2\sin(2t - 16)$

$g'(8) = e > 0$

$g''(t) = \frac{1}{7}e^{\frac{1}{7}(t-1)} - 4\cos(2t - 16)$

$g''(8) = \frac{1}{7}e - 4 < 0$

$g'(8) > 0$ and $g''(8) < 0$, therefore at $t = 8$, the number of bees is increasing at a decreasing rate.

$g(t)$ is concave down at $t = 8$ since $g''(8) < 0$.

Score:

1: increasing at a decreasing rate since $g'(8) > 0$ and $g''(8) < 0$

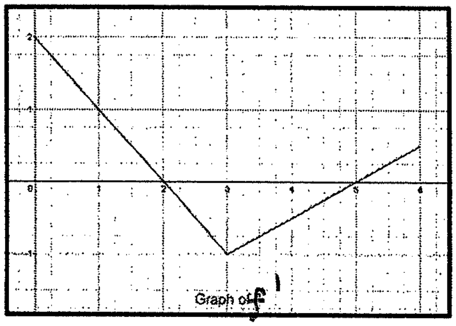
1: g is concave down at $t = 8$, with reason

(e) The number of bees spotted in Amelie's garden can also be modeled by the function $B(x) = 50\sqrt{k + 2x}$ where x is the daily high temperature, in degrees Farenheit, and k is a positive constant. When the number of bees spotted is 100, the daily high temperature is increasing at a rate of 2°F per day. According to this model, how quickly is the number of bees changing with respect to time when 100 bees are spotted?

AB Calculus Name Key
2020 Exam Practice
FR #4 (25 minutes: 15 points)

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

Both $f(x)$ and $g(x)$ are differentiable on the interval [0,6]. It is known that $f(0) = -2$. The graph of f', consisting of 2 line segments, is shown below. A table of values for $g(x)$, $g'(x)$, and $g''(x)$ is given.



x	**0**	**2**	**4**	**6**
g(x)	4	0	2	-2
g'(x)	-4	0	0	-4
g''(x)	2	2	-2	-2

Grading Rubric

a) At what value of x does $\boxed{f(x) \text{ have a relative maximum}}$? Give a reason. [2 points]

(Max & min) occurs where $f'(x) = 0$

$f'(x) = 0$ at $x = 2, 5$.

$f(x)$ has a rel. max at $x = 2$ because $f'(x)$ changes from positive to neg. at that value

1: Identifies $x = 2$

1: correct reason

b) At what value of x does $\boxed{g(x) \text{ have a relative minimum?}}$ Give a reason. [2 points]

HINT
Need to look at $g'(x)$ and $g''(x)$.

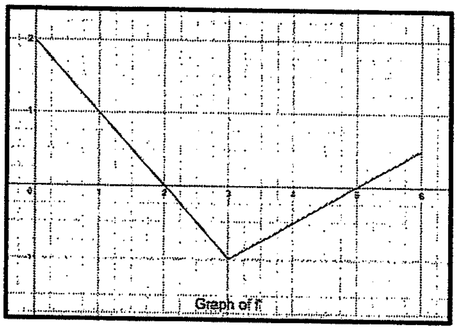
$g(x)$ has a rel. min at $x = 2$ because $g'(2) = 0$ and $g''(2) = 2$. Since $g''(2) > 0$, this means that $g(x)$ is concave up at this point. This is the 2nd derivative test.

(not necessary to name, but good to know)

1: Identifies $x = 2$

1: reason

Both $f(x)$ and $g(x)$ are differentiable on the interval [0,6]. It is known that $\boxed{f(0) = -2.}$ The graph of f', consisting of 2 line segments, is shown below. A table of values for $g(x)$, $g'(x)$, and $g''(x)$ is given.



x	0	2	4	6
g(x)	4	0	2	-3
g'(x)	-4	0	0	-4
g''(x)	2	2	-2	-2

starting location + area

Grading Rubric

c) Write an expression for $f(x)$ that includes an integral. Use that expression to find the values of $f(2)$ and $f(5)$. [3 points]

$$\boxed{f(x) = -2 + \int_0^x f'(t)\,dt}$$

$$f(2) = -2 + \int_0^2 f'(t)\,dt = $$

$$= -2 + \left(\tfrac{1}{2}(2)(2)\right)$$

$$\boxed{f(2) = 0}$$

$$f(5) = -2 + \int_0^5 f'(t)\,dt$$

$$= -2 + \left[\tfrac{1}{2}(2)(2)\right] - \left[\tfrac{1}{2}(3)(1)\right]$$

$$\boxed{f(5) = -\frac{3}{2}}$$

1: f(x) expression

1: $f(2) = 0$

1: $f(5) = -\frac{3}{2}$

d) Find the absolute minimum value of $f(x)$ on the interval [0,6]. Show the analysis that leads to your answer. [3 points]

HINT: Must include endpoints ☺

Test endpoints and c.p's.

c.p's from (a) are $x = 2, 5$.

Endpoints are $x = 0, 6$.

$f'(x)$: + on 0 to 2, − on 2 to 5, + on 5 to 6

test: $f(0) = -2$

$f(5) = -\frac{3}{2}$

So abs min occurs at $x = 0$ and is -2

1: uses c.p's and endpoints

1: finds $x = 0$

1: value of -2

e) On the interval [0,6], is there a value of c such that $g'(c) = -2$? If so, explain your reasoning. If not, state that there is no such value. [2 points]

Yes.

Given: g(x) is differentiable, therefore continuous.

$$\text{So } \frac{g(2) - g(0)}{2 - 0} = \frac{0 - 4}{2 - 0} = -2$$

By the MVT, if a fn is continuous & differentiable, then there must be a value where the der. equals the average. [redacted]

1: Uses MVT correctly

1: Correct points used for ave

f) Find the value of this limit: $\lim_{x\to 2} \frac{f(x)}{g(x)}$. Show all supporting work. [3 points]

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \frac{f(2)}{g(2)} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{f(x)}{g(x)}$$

$$\lim_{x\to 2} \frac{f'(2)}{g'(2)} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 2} \frac{f''(2)}{g''(2)} = \boxed{\frac{-1}{2}}$$

1: Tries direct substitution $\left(\frac{0}{0}\right)$

1: Use L'Hopital's rule correctly

1: Correct answer

AP Calculus
Ricketts

Free Response Question – G1
Time guideline – 2 problems within 30 minutes

Name: Key

A graphing calculator is required for these problems.

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $\boxed{G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)}$, where t is measured in hours and $\boxed{0 \le t \le 8}$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Grading Rubric

Rate of gravel arrival $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right); 0 \le t \le 8$

$(0, 500)$

Processes gravel rate 100 tons/hr.

a) $G'(5) = G'(t) = -45\sin\left(\frac{t^2}{18}\right) \cdot \frac{1}{18} 2t$

$G'(5) = -45\sin\left(\frac{25}{18}\right) \cdot \frac{1}{18}(2 \cdot 5)$

$= -24.587$ tons/hr^2 ①

The rate at which gravel is arriving is decreasing by 24.587 tons/hour per hour at t = 5 hrs. ①

$2\left\{\begin{array}{l} 1: G'(5) \\ 1: \text{intrepretation with units} \end{array}\right.$

b) Total amt is $\int_0^8 G(t)\,dt = \int_0^8 \left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)dt$ ①

$= 825.551$ tons ①

$2\left\{\begin{array}{l} 1: \text{integral} \\ 1: \text{answer} \end{array}\right.$

c). Compare the rate of arrival to the rate of processing.

$G(5) = 90 + 45\cos\left(\frac{5^2}{18}\right)$ Rate of processing

$= 98.140 < 100$ ①

↓

rate of arrival of gravel.

1: compares G(5) to 100

2

1: conclusion

Justification At time t = 5, the rate of gravel arriving is less than the rate of processing. ① So the amt of unprocessed gravel is decreasing at time = 5.

d). $\text{Amt}(t) = \text{beg amt} + \int_0^t (G(x) - 100)\,dx$

↓

(arrival rate − processing rate)

$A(t) = 500 + \int_0^t (G(x) - 100)\,dx$.

To find max/min, take derivative to find c.p's. Test c.p's and endpoints to find t at which max occurs.

$A'(t) = \frac{d}{dx}\int_0^t (G(x) - 100)\,dx$ ①

$= G(t) - 100 \Rightarrow \text{set} = 0.$ (Graph)

$t = 4.9234803$ Ⓑ, endpoints 0, 8.

3

1: sets A'(t)=0

1: answer

1: justification

t	A(t)
0.	500
4.9.... Ⓑ	$500 + \int_0^B (G(x) - 100)\,dx$ $= 635.3761231$
8	$500 + \int_0^8 (G(x) - 100)\,dx$ $= 525.5510886$

①

Justify The max amount of unprocessed gravel is 635.376 tons which occurs at t = 4.923 hrs. ①

AP Calculus
Ricketts

Free Response Question – G2B
Time guideline – 2 problems within 30 minutes

Name: Key.

A graphing calculator is required for these problems.

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

Grading Rubric

a) speed = |velocity| ; $2 \le t \le 4$

graph $y_1 = |v(t)|$ ①
$y_2 = 2$.

$t = 3.1276299$ ①
$t = 3.473402$.

2 { 1: considers $|v(t)| = 2$
1: answer

b). $s(t) = 10 + \int_0^t v(x)\,dx$ ①

$\underbrace{10}_{\text{starting position}}$ — $\int_0^t v(x)\,dx$: integral of velocity = position.

$s(5) = 10 + \int_0^5 v(t)\,dt$
$= -9.207$. ①

2 { 1: s(t)
1: s(5)

c) Particle changes direction when v(t) goes from pos → neg or vice versa. ①
Find where $v(t) = 0$.

$v(t) = 0$ at $t = 3.3177563$ pos → neg.
" " at $t = 0.53603315$ neg → pos. ②

3 { 1: considers $v(t) = 0$
2: answers with justification

d) Speed increasing means velocity and acceleration have the same sign.

Speed decreasing is when velocity and acceleration

$v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$

$a(t) = \left(\frac{6}{5}(t^2 + 3t)^{1/5} \cdot (2t + 3)\right) - 3t^2$

$v(4) = -2 + (4^2 + 3(4))^{6/5} - 4^3$

$= -11.475758$

①

$a(4) = \left(\frac{6}{5}(4^2 + 3(4))^{1/5} \cdot (11)\right) - 3(4)^2$

$= -22.295714.$

Speed is increasing at t = 4 because both velocity and acceleration have the same sign. ①

2: conclusion and reasoning

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[redacted] SCORING GUIDELINES

Question 3

G3B

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$$

(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval)

$\dfrac{C(4) - C(2)}{4 - 2} = \dfrac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$$

(c)
$$\begin{aligned} \frac{1}{6}\int_0^6 C(t)\,dt &\approx \frac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)] \\ &= \frac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8) \\ &= \frac{1}{6}(60.6) = 10.1 \text{ ounces} \end{aligned}$$

$\frac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

$$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$$