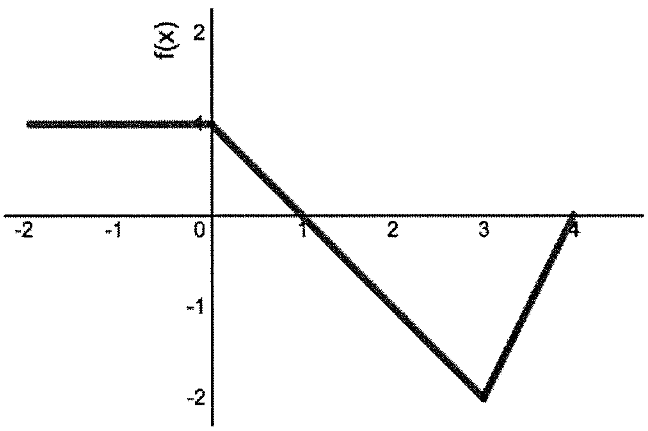
Circuit Training – Fundamental Theorem of Calculus, Part I   Name ________________

Directions: Beginning in cell #1, use the Fundamental Theorem of Calculus Part I (and occasionally Part II) to answer the question. Search for your answer and that problem becomes #2. Continue in this manner until you complete the circuit.
NOTE: Any questions about the function $H(t)$ pertain to the following given information...

Let $H(t) = \int_1^t f(x)dx$ where $f(x)$ is the continuous function composed of three line segments with domain $[-2, 4]$ as graphed below:



$H(t) = \int_1^t f(x)\,dx$
$H'(t) = f(t)$
$H''(t) = f'(t)$

$\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$
$\frac{3}{2}\pi$ -1
$\sec^2\frac{\pi}{4} = \frac{2}{\sqrt{2}}$

<table>
<tr>
<td>Answer: $-\frac{5}{2}$ ✓

__1__ Let $F(x) = \int_0^x 5dt$. Find $F(3)$.

$F(3) = 5x\Big]_0^3$
$= 5(3) - 5(0)$
$= 15$</td>
<td>Answer: $\frac{2}{3}$ ✓

__8__ Find $\frac{d}{dt}\int_{-1}^{\tan t} \frac{2}{1+x^2}dx$ and evaluate it for $t = -\frac{\pi}{4}$.

$\frac{d}{dt}\int_{-1}^{\tan t} \frac{2}{1+x^2} = \frac{2}{1+(\tan t)^2} =$
$t = -\frac{\pi}{4} \Rightarrow \frac{2}{1+(\tan -\frac{\pi}{4})^2} = \frac{2}{1(-1)^2} = \frac{2}{1}$
$= 2$</td>
</tr>
<tr>
<td>Answer: 1 ✓
__5__
Find $F'\left(\frac{3\pi}{2}\right)$ given $F(t) = \int_5^t \frac{2x}{\pi}e^{\cos x}dx$.

$F'(t) = \frac{d}{dt}\left(\int_5^t \frac{2x}{\pi}e^{\cos x}dx\right)$
$= \frac{2t}{\pi}e^{\cos t}\,dt$
$F'\left(\frac{3\pi}{2}\right) = \frac{2}{\pi}\left(\frac{3\pi}{2}\right)e^{\cos\left(\frac{3\pi}{2}\right)} = 3e^0 = 3.$</td>
<td>Answer: – 2 ✓

__11__ $H''(1) = ?$

$m = \frac{-2-1}{3-0} = \frac{-3}{3} = -1$
$H''(1) = f'(1)$
$= -1$</td>
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</table>

Answer: -1

\# 12 $H(-2) = ?$

$H(t) = \int^{t} f(x)\,dx$

$H(-2) = -\left[2 + \frac{1}{2}\right]$

$= -\frac{5}{2}$

Answer: 4

\# 4

$$G(x) = \int_{-2}^{x} \cos\left(\theta + \tfrac{\pi}{2}\right) d\theta. \quad G'\left(-\tfrac{\pi}{2}\right) =$$

$G'(x) = \cos\left(x + \frac{\pi}{2}\right)$

$G'\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$

$= \cos 0$

$= 1$

Answer: 15

\# 2

Let $G(x) = \displaystyle\int_{x}^{2} t\,dt$. Find $G(4)$.

$G(x) = -\int_{2}^{x} t\,dt$

$G(4) = -\left(\frac{1}{2}t^2\right)\Big|_2^4$

$= -\left(\frac{1}{2}(4)^2 - \frac{1}{2}(2)^2\right) = -6$

Answer: -3

\# 7

Given $W(t) = \int_2^t \ln(x-1)\,dx$. Find $W''\left(\frac{5}{2}\right)$.

$W'(t) = \frac{d}{dt}\int_2^t \ln(x-1)\,dx$

$= \ln(t-1)$

$W''(t) = \frac{1}{t-1} \cdot 1$

$W''\left(\frac{5}{2}\right) = \frac{1}{\frac{5}{2}-1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

Answer: 3

\# 6

The position function, $s(t)$, is defined as $s(t) = s(0) + \int_0^t \left(\frac{8}{\pi} + \sec^2\beta\right) d\beta$ where $s(0) = -6$. Find $s\left(\frac{\pi}{4}\right)$.

$S(t) = S(0) + \left[\frac{8}{\pi}t + \sec^2 t\right]_0^t$

$S\left(\frac{\pi}{4}\right) = -6 + \left[\frac{8}{\pi}\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\right]$

$= -6 + (2+1) - 0$

$= -6 + 3$

$= -3$

Answer: 0

\# 10 Now evaluate $H'(3)$.

$H'(t) = f(t)$

$H'(3) = f(3)$

$= -2$

Answer: 2

\# 9 The next questions are about $H(t)$. Evaluate $H(1)$.

$H(t) = \int^{t} f(x)\,dx$

$H(1) = \int_1^1 f(x)\,dx$

$= 0$

Answer: -6

\# 3

Let $F(x) = \int_3^x \sqrt{1+t}\,dt$. Find $F'(15)$.

$F'(x) = \frac{d}{dx}\left(\int_3^x \sqrt{1+t}\,dt\right)$

$= \sqrt{1+x}$

$F'(15) = \sqrt{1+15}$

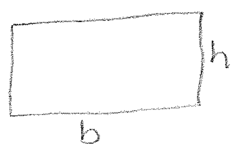
$= 4$

Circuit Training – Related Rates Name Key

Directions: Beginning in cell #1, read the problem, identify the key information, and sketch a helpful picture. Decide what formula relates all of your information, and differentiate it with respect to t. Finally, answer the question and search for it to advance in the circuit. Mark that cell #2 and continue in this manner until you complete the circuit. Note: Technology should be used in the final stages of solving. Round all answers to three decimal places, but hold intermediate values to at least five decimal places.

Answer: 1.185

1 A rectangle's base remains 0.5 cm while its height changes at a rate of 1.5 cm/min. At what rate is the area changing, in cm^2/min, when the height is 1.5 cm?



Known rate $\frac{dh}{dt} = 1.5 cm/min$

Unknown rate $\frac{dA}{dt}$

$A = bh$

$A = 0.5h$

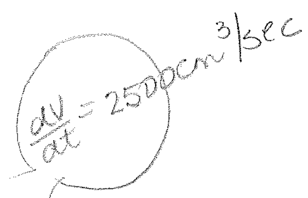
$\frac{dA}{dt} = 0.5\frac{dh}{dt}$

$= 0.5(1.5)$

$\boxed{\frac{dA}{dt} = 0.750 cm^2/min}$

Answer: 1.035

6 A spherical balloon is losing air at a rate of 2500 cm^3/sec. What is the radius, in cm, when it is changing at a rate of 25 cm/sec?



Also $\frac{dr}{dt} = 25 cm/sec$

Find r

$V(sphere) = \frac{4}{3}\pi r^3$

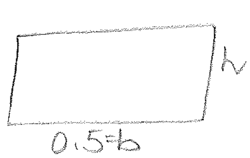
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2500 = 4\pi r^2 (25)$

$\boxed{2.820 \text{ cm} = r}$

Answer: 0.750

2 A rectangle's base remains 0.5 cm while its height changes at a rate of 1.5 cm/min. At what rate is the perimeter changing, in cm/min, when the height is 1.5 cm?



$\frac{dh}{dt} = 1.5 cm/min$

Unknown rate $= \frac{dP}{dt}$

$P = 2b + 2h$

$P = 2(0.5) + 2h$

$P = 1 + 2h$

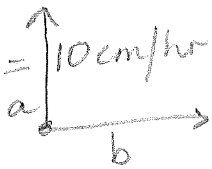
$\frac{dP}{dt} = 2\frac{dh}{dt}$

$\boxed{\frac{dP}{dt} = 3 cm/min}$

Answer: 4.909

9 Two snails start moving away from each other from the same point in a garden. One travels north at a rate of 10 cm/hour and the other travels east at a rate of 3 cm/hour. After two hours, how fast is the distance between the snails changing, in cm/hour?

$\frac{da}{dt} =$



$\frac{db}{dt} = 3\text{cm/hr}$

Find rate $\frac{dc}{dt}$

Distance = rate × time

$\therefore a = 10(2) = 20\text{cm}$

$b = 3(2) = 6\text{ cm}$

$c = \sqrt{a^2+b^2}$

$= \sqrt{436}\text{cm}.$

$a^2+b^2=c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

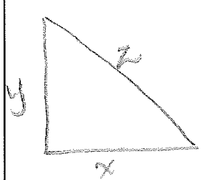
$\therefore a\frac{da}{dt} + b\frac{db}{dt} = c\frac{dc}{dt}$

$20(10) + 6(3) = \sqrt{436}\frac{dc}{dt}$

$\boxed{10.440\text{cm/hr} = \frac{dc}{dt}}$

Answer: 3.000

3 A right triangle has legs x and y and hypotenuse z. At what rate is leg x changing, in cm/min, when y = 4 cm, dy/dt = 1.2 cm/min, z = 5 cm, and dz/dt = 3.4 cm/min?



Find $\frac{dx}{dt}$

Know $\frac{dy}{dt} = 1.2\text{cm/min}$

$\frac{dz}{dt} = 3.4\text{cm/min}$

Find x: $x^2+y^2=z^2$

$x=3$

Pythagorean triple

$x^2+y^2=z^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

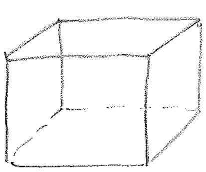
$3\left(\frac{dx}{dt}\right) + 4(1.2) = 5(3.4)$

$3\frac{dx}{dt} = 12.2$

$\boxed{\frac{dx}{dt} = 4.067\text{ cm/min}}$

Answer: 10.440

10 The volume of a cube is 125 cm³ and is changing at a rate of 8 cm³/sec. How fast is the surface area changing, in cm²/sec, at that moment in time?



$\frac{dV}{dt} = 8\text{cm}^3/\text{sec}$

Need $\frac{d(SA)}{dt}$

$V=s^3$

$125=s^3$

$5=s$

$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$

$8 = 3(5)^2\frac{ds}{dt}$

$0.1066\ldots = \frac{ds}{dt}$

$SA = 6s^2$

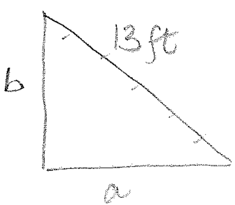
$\frac{d(SA)}{dt} = 12s\frac{ds}{dt}$

$\frac{d(SA)}{dt} = 12(5)(0.1066..)$

$\boxed{\frac{d(SA)}{dt} = 6.4\text{cm}^2/\text{sec}}$

Answer: 4.067

\# 4 A 13-foot ladder slides down an exterior wall at a rate of 1 ft/sec. How fast is the distance between the base of the ladder and the base of the wall changing, in ft/sec, when the base of the ladder is 5 ft from the base of the wall?



$a^2+b^2=c^2$

$5^2+b^2=13^2$

$b=12$

$a^2+b^2=c^2$

$a\frac{da}{dt}+b\frac{db}{dt}=0$

$5\frac{da}{dt}=-12(-1)$

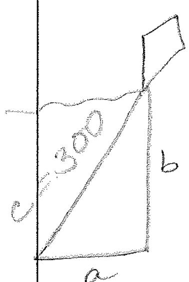
$\boxed{\frac{da}{dt}=2.400\text{ ft/sec}}$

$\frac{db}{dt}=-1$ ft/sec

Find $\frac{da}{dt}$

Answer: 2.821

\# 7 Charlie Brown flies a kite at a constant height of 100 feet while the wind carries the kite away from him horizontally at rate of 2 ft/sec. How fast must he release the string from the spool, in ft/sec, when the kite is 300 feet away from him (i.e. when he has let out 300 feet of string)?



$a^2+b^2=c^2$

$a^2+100^2=300^2$

$a=282.8427\ldots$

$a^2+b^2=c^2$

$2a\frac{da}{dt}+0=2c\frac{dc}{dt}$

$a\frac{da}{dt}=c\frac{dc}{dt}$

$282.847\ldots(2)=300\left(\frac{dc}{dt}\right)$

$\boxed{1.885\text{ ft/sec}=\frac{dc}{dt}}$

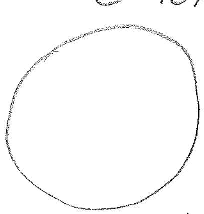
Rate Known: $\frac{da}{dt}=2$ ft/sec

Find $\frac{dc}{dt}$

Answer: 2.400

\# 5 A circle is expanding at a rate of 32.5 cm^2/min. How fast is the radius changing, in cm/min, when the circumference is 10π cm?

$C=10\pi$



$C=2\pi r$

$10\pi=2\pi r$

$5=r$

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

$32.5=2\pi(5)\frac{dr}{dt}$

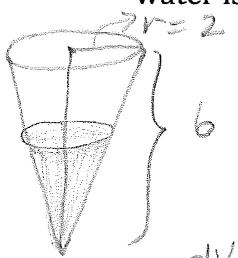
$\boxed{1.034\text{ cm/min}=\frac{dr}{dt}}$

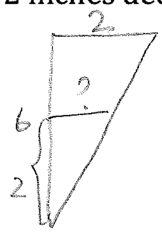
Known $\frac{dA}{dt}=32.5$ cm^2/min

Find $\frac{dr}{dt}$

Answer: 6.400

11 A conical paper drinking cup has a height of 6 inches and a radius of 2 inches (at the top). How fast is the water level in the cup rising if water is poured in at a rate of 1 in³/sec and the water is 2 inches deep?





$\frac{2}{6} = \frac{r}{h}$

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{27}\pi h^3$

Known $\frac{dV}{dt} = 1 \text{ in}^3/\text{sec}$

Find $\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$1 = \frac{1}{9}\pi (2)^2 \frac{dh}{dt}$

$1 = \frac{4\pi}{9} \frac{dh}{dt}$

$\boxed{0.716 \text{ in/sec} = \frac{dh}{dt}}$

Answer: 0.716

12 Profit is Revenue – Cost. A manufacturing plant calculates its Cost function, C(x), to be $C(x) = x^3 - 4x^2 + 50/x$ and its Revenue function, R(x), to be R(x) = 50x. Construct the profit function, P(x), and determine the x-value when dP/dt is 4.544 and dx/dt = 0.05.

$P(x) = R(x) - C(x)$

$P(x) = 50x - (x^3 - 4x^2 + 50x^{-1})$

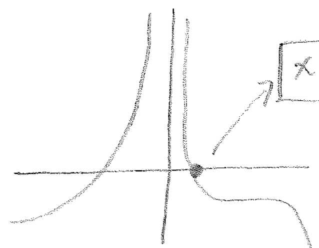
$\frac{dP}{dt} = \left(50 - 3x^2 + 8x + \frac{50}{x^2}\right)\frac{dx}{dt}$

$4.544 = \left(50 - 3x^2 + 8x + \frac{50}{x^2}\right)(0.05)$

$4.544 = 2.5 - 0.15x^2 + 0.4x + 2.5x^{-2}$

$0 = -2.044 - 0.15x^2 + 0.4x + 2.5x^{-2}$

Graph & solve for x



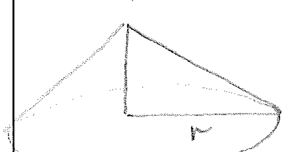
$\boxed{x = 1.1849}$

Answer: 1.886

8 Salt for winter road maintenance is poured so that it forms a conical pile such that the radius is twice the height. Find the rate at which the pile is growing, in ft³/hour, when the radius is changing at a rate of 0.5 ft/hour and measures 2.5 feet.

Know $\frac{dr}{dt} = 0.5$

Find $\frac{dV}{dt}$



$r = 2h$

$\frac{1}{2}r = h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2\left(\frac{1}{2}r\right)$

$V = \frac{1}{6}\pi r^3$

$\frac{dV}{dt} = \frac{1}{2}\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{1}{2}\pi (2.5)^2 (0.5)$

$\boxed{\frac{dV}{dt} = 4.908 \text{ ft}^3/\text{hr}}$

P.S. Of course I know how unlikely these crazy questions are! You have to practice them anyway!