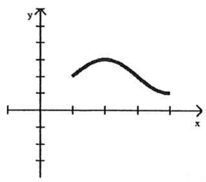
**Fundamental Theorem of Calculus – Part II** (NC) **Name:** Key

**Write out the FTC II (in words), then use it to solve the problems that follow.**

If a fn. is continuous on closed [a,b] and has an antiderivative, then the definite integral is the difference of its antiderivatives at the upper & lower bounds.

1. The graph of $f'$ is shown below. $\int_1^4 f'(x)dx = 6.2$ and $f(1) = 3$. Find $f(4)$.



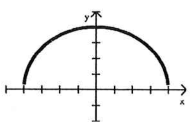
**Show work here:**

$f(4) = f(1) + \int_1^4 f(x)dx$

$= 3 + 6.2$

$\boxed{f(4) = 9.2}$

2. The graph of $f'$ is the semi-circle below. Find $f(-4)$ given that $f(4) = 7$.



$f(-4) = f(4) + \int_4^{-4} \frac{\pi r^2}{2} dr$

$= f(4) - \frac{\pi}{2}\int_{-4}^{4} r^2 dr$

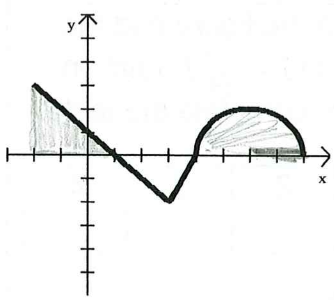
$r = 4$

$= 7 - \frac{16\pi}{2}$

$\boxed{f(-4) = 7 - 8\pi}$

3. The graph of $f'$, consisting of two line Segments and a semicircle, is shown. Given that $f(-2) = 5$, find:

(a) $f(1)$ (b) $f(4)$ (c) $f(8)$



$r = 2$

a) $f(1) = f(-2) + \int_{-2}^{1} f(x)dx$

$= 5 + \frac{9}{2}$

$\boxed{f(1) = 9.5}$

b) $f(4) = f(-2) + \int_{-2}^{1} f(x)dx + \int_{1}^{4} f(x)dx$

$= \underbrace{f(-2) + \int_{-2}^{1} f(x)dx}_{9.5} + (-3)$

$\boxed{f(4) = 6.5}$

c) $f(8) = f(-2) + \int_{-2}^{4} f(x)dx + \int_{4}^{8} \frac{\pi r^2}{2} dr$

$= \underbrace{f(-2) + \int_{-2}^{4} f(x)dx}_{6.5} + 2\pi$

$\boxed{f(8) = 6.5 + 2\pi}$

$\frac{81}{9}$ $\frac{729}{3} = 243$

4. $\int_0^9 (e^x - x^2)dx$

$F(x) = e^x - \frac{1}{3}x^3 \Big|_{x=0}^{x=9}$

$F(x) = \left(e^9 - \frac{1}{3}(9)^3\right) - \left(e^0 - \frac{1}{3}(0)^3\right)$

$= e^9 - 243 - 1$

$\boxed{F(x) = e^9 - 242}$

5. $\int_1^9 (t - \sqrt{t})dt = \int^9 (t - t^{1/2})dt$

$F(x) = \frac{1}{2}t^2 - \frac{2t^{3/2}}{3}\Big]_{t=1}^{t=9}$

$F(x) = \left[\frac{1}{2}(9)^2 - \frac{2(9)^{3/2}}{3}\right] - \left[\frac{1}{2}(1)^2 - \frac{2(1)^{3/2}}{3}\right]$

$= \left(\frac{81}{2} - 18\right) - \left(\frac{1}{2} - \frac{2}{3}\right)$

$= 22.5 - \left(-\frac{1}{6}\right)$

$\boxed{F(x) = \frac{68}{3} \text{ or } 22\frac{2}{3}}$

6. $\int_4^{16} \left(\frac{p-1}{p}\right) dp = \int_4^{16} \left(1 - \frac{1}{p}\right)dp$

$F(x) = p - \ln|p| \Big]_{p=4}^{p=16}$

$F(x) = [16 - \ln|16|] - [4 - \ln|4|]$

$= 12 - \ln 16 + \ln 4$

$= 12 + \ln \frac{4}{16}$ $\quad 12 - \ln 4$

$= 12 + \ln \frac{1}{4}$ $\quad \boxed{F(x) = 12 - \ln 4}$

7. $\int_0^5 \frac{1}{(x-3)^2} dx$ (Watch out)!!

$f(x) = \frac{1}{(x-3)^2}$ is not continuous over the interval $[0,5]$ at $x=3$.

$\therefore \int_0^5 \frac{1}{(x-3)^2} = \text{undefined}.$

8. $\int_0^1 \frac{1}{(x-3)^2} dx = \int_0^1 (x-3)^{-2} dx.$

$F(x) = -\frac{1}{x-3}\Big]_{x=0}^{x=1}$

$F(x) = \left[\frac{-1}{1-3} - \frac{-1}{0-3}\right]$

$F(x) = \frac{1}{2} - \frac{1}{3}$

$\boxed{F(x) = \frac{1}{6}}$

9. Use the function $f$ in the figure and the function $F$ defined by $F(x) = \int_0^x f(t)dt$ on the interval $0 \le x \le 4$ to complete the table.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $F(x)$ | 0 | 1 | 1 | 1/4 | 0 |

$F(x)$ = area under the curve thus far.