AP Calculus
Ricketts

Fundamental Theorem of Calculus – Part I (NC) **Name:** Key

Write out the FTC I (in words), then use it to solve the problems that follow.

If a fn. is continuous on a closed interval and differentiable, then the derivative of an integral is the function evaluated at its upper limit

For each problem, find $F'(x)$. Be accurate with notation, especially with u –substitution.

1. $F(x) = \int_{-4}^{x}(t-1)dt$

$\boxed{F'(x) = x - 1}$

2. $F(x) = \int_{-3}^{x}(t^2+2t+3)dt$

$\boxed{F'(x) = x^2 + 2x + 3}$

3. $F(x) = \int_{-1}^{x^2}(-2t+2)dt$

Let $u = x^2$, $\frac{du}{dx} = 2x$

$F'(u) = \left(\frac{d}{du}\left(\int_{-1}^{u}(-2t+2)dt\right)\frac{du}{dx}\right)$

$= (-2u+2)\frac{du}{dx}$

$F'(x) = (-2(x^2)+2)\cdot 2x$

$\boxed{F'(x) = -4x^3 + 4x}$

4. $F(x) = \int_{4}^{3x}(-t^3+11t^2-39t+44)dt$

Let $u = 3x$, $\frac{du}{dx} = 3$.

$F'(u) = \left(\frac{d}{du}\int_{4}^{u}(-t^3+11t^2-39t+44)dt \cdot \frac{du}{dx}\right)$

$= (-u^3+11u^2-39u+44)\cdot 3$

$F'(x) = \left[-(3x)^3+11(3x)^2-39(3x)+44\right]3$

$\boxed{F'(x) = -81x^3 + 297x^2 - 351x + 132}$

5. $F(x) = \int_{2}^{x^3}\left(\frac{1}{t^3}\right)dt$

Let $u = x^3$, $\frac{du}{dx} = 3x^2$

$F'(u) = \left(\frac{d}{du}\int^{u}\left(\frac{1}{t^3}\right)dt\right)\frac{du}{dx}$

$= \frac{1}{(u^3)^3} \cdot \frac{du}{dx}$

$F'(x) = \frac{1}{x^9} \cdot 3x^2$

$\boxed{F'(x) = \frac{3}{x^7}}$

6. $F(x) = \int_{1}^{x^3} t\sin(2t)dt$

Let $u = x^3$, $\frac{du}{dx} = 3x^2$

$F'(u) = \left(\frac{d}{du}\int_{1}^{u}(t\sin(2t)\,dt) \cdot \frac{du}{dx}\right)$

$= u\sin 2u \cdot \frac{du}{dx}$

$F'(x) = (x^3\sin(2x^3))3x^2$

$\boxed{F'(x) = 3x^5\sin(2x^3)}$

Answers: $F'(x) = \frac{3}{x^7}$ $F'(x) = -4x^3 + 4x$ $F'(x) = 3x^5\sin(2x^3)$

$F'(x) = -81x^3 + 297x^2 - 351x + 132$ $F'(x) = x^2 + 2x + 3$ $F'(x) = x - 1$