AP Calculus
Ricketts

Key

CHAPTER 5 – Additional Notes

Riemann sum to Integral & Integral to Riemann Sum

A. Write the general form of the limit of a Riemann sum below:

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=1}^{n} \underbrace{f\left(\underbrace{a+\left(\frac{b-a}{n}\right)k}_{x}\right)}_{f(x)} \underbrace{\left(\frac{b-a}{n}\right)}_{dx}$$

B. From an integral to the limit of Riemann sums. lower limit

a) $\int_0^3 (3x-8)dx$ $= \lim_{n \to \infty} \sum_{k=1}^{n} \left[\underbrace{3}_{3}\left(\underbrace{0+\frac{3}{n}k}_{x}\right)\underbrace{-8}_{-8}\right]\underbrace{\left(\frac{3}{n}\right)}_{dx}$

$a = 0$ $\quad \frac{b-a}{n}$

$\Delta x = \frac{3-0}{n} = \frac{3}{n}$ $\quad = \lim_{n \to \infty} \sum_{k=1}^{n} \left[3\left(\frac{3}{n}k\right)-8\right]\left(\frac{3}{n}\right)$

b) $\int_{-1}^2 (2x)dx$ $= \lim_{n \to \infty} \sum_{k=1}^{n} \left[\underbrace{2}_{2}\left(\underbrace{-1+\frac{3}{n}k}_{x}\right)\right]\underbrace{\left(\frac{3}{n}\right)}_{(dx)}$

$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$

C. From a Riemann sum to an integral. Each c_k is chosen from the kth subinterval of a regular partition of the indicated interval into n subintervals of length Δx.

a) $\lim_{n \to \infty} \sum_{k=1}^{n} (3c_k + 10)\Delta x; \quad [-1, 5]$

$$= \int_{-1}^{5} (3x+10)\,dx$$

b) $\lim_{n \to \infty} \sum_{k=1}^{n} \sqrt{c_k^2 + 4}\,\Delta x; \quad [0, 3]$

$$= \int_0^3 \left(\sqrt{x^2+4}\right) dx$$