Precalculus Name:

Exam Review Chapters 5,6,7, 9,10

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the fundamental identities to find the value of the trigonometric function.

1) Find $\tan\theta$ if $\cos\theta = \frac{1}{6}$ and $\sin\theta < 0$.

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{1}{6}\right)^2 = 1$ → $\sin^2\theta = 1 - \frac{1}{36}$, $\sin^2\theta = \frac{36}{36} - \frac{1}{36}$, $\sin\theta = -\sqrt{\frac{35}{36}}$, $\sin\theta = -\frac{\sqrt{35}}{6}$

$\tan\theta = -\sqrt{3}$

2) Find $\cot\theta$ if $\csc\theta = \frac{\sqrt{37}}{6}$ and $\tan\theta > 0$.

$1 + \cot^2\theta = \csc^2\theta \rightarrow 1 + \cot^2\theta = \frac{37}{36} \rightarrow \cot^2\theta = \frac{1}{36} \rightarrow \cot\theta = \frac{1}{6}$

Use basic identities to simplify the expression.

3) $\cot\theta \sec\theta \sin\theta$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \cdot \sin\theta = 1$

4) $\sin\theta \cos\theta \sec\theta \csc\theta$ → $\sin\theta \cos\theta \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = 1$

5) $\frac{\tan\theta}{\cot\theta}$ $\frac{\tan\theta}{\frac{1}{\tan\theta}} \rightarrow \tan\theta \cdot \tan\theta = \tan^2\theta$

Find all solutions in the interval $[0, 2\pi)$.

6) $\cos^2 x + 2\cos x + 1 = 0$

$(\cos x + 1)(\cos x + 1) = 0$, $\cos x = -1$ → $x = \pi$

7) $2\sin^2 x = \sin x$

$-\sin x \quad -\sin x$

$2\sin^2 x - \sin x = 0$ → $\sin x(2\sin x - 1) = 0$

$\sin x = 0$, $x = 0, \pi$

$2\sin x - 1 = 0$, $\sin x = \frac{1}{2} \rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

Prove the identity.

8) $\cos x \csc x \tan x = 1$

$\cos x \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$ (Reciprocal/Quotient properties)

$= 1$

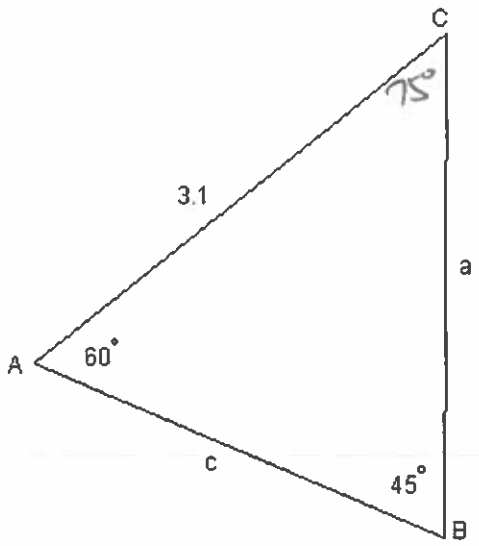
9) $\frac{1 - \sin t}{\cos t} = \frac{\cos t}{1 + \sin t}$ $\frac{(1-\sin t)}{(1-\sin t)}$

$= \frac{\cos t(1-\sin t)}{1-\sin^2 t}$

$= \frac{\cos t(1-\sin t)}{\cos^2 t} = \frac{1-\sin t}{\cos t}$ ∎.

Solve the triangle.

10)



$\frac{\sin 45}{3.1} = \frac{\sin 60}{a}$

$a \approx 3.797$

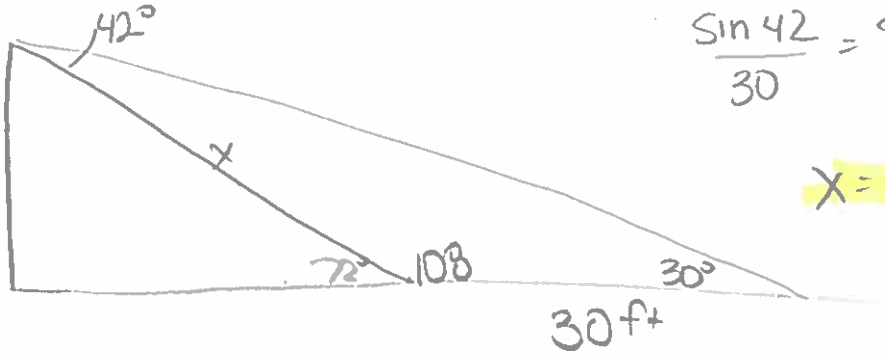
$\angle C = 75°$

$\frac{\sin 45°}{3.1} = \frac{\sin 75}{c}$

$c \approx 4.235$

Solve.

11) A guy wire to a tower makes a 72° angle with level ground. At a point 30 ft farther from the tower than the wire but on the same side as the base of the wire, the angle of elevation to the top of the tower is 30°. Find the length of the wire (to the nearest foot).



$\frac{\sin 42}{30} = \frac{\sin 30}{x}$

$x = 22$ ft.

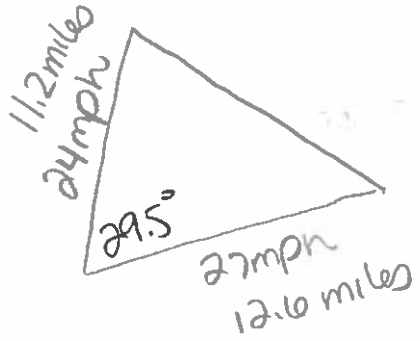
Solve the problem.

12) Two boats leave a dock together, each traveling in a straight line. One boat travels at 27 mph and the other at 24 mph. If the angle between their courses measures 29.5°, how far apart are they after 28 minutes? Give your answer in miles and round your answer to the nearest tenth.

28 mins = .46 hours

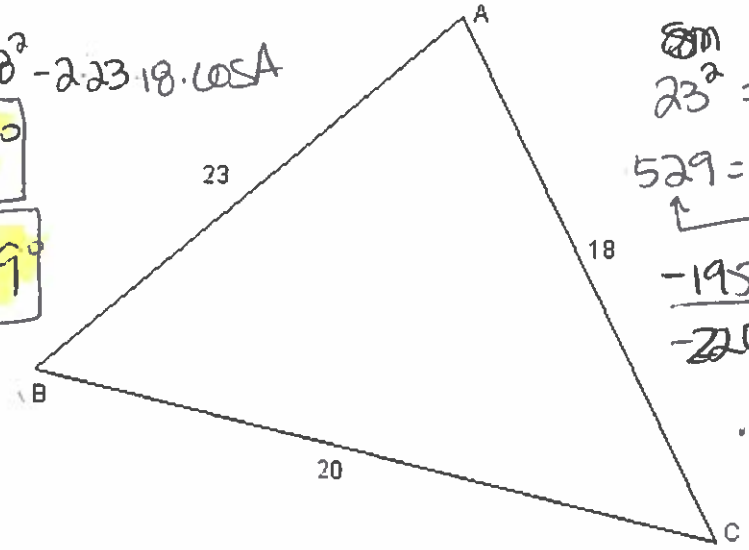
$c^2 = 11.2^2 + 12.6^2 - 2 \cdot 11.2 \cdot 12.6 \cdot \cos 29.5°$

6.2 miles



Solve the triangle.

13)



$20^2 = 23^2 + 18^2 - 2 \cdot 23 \cdot 18 \cdot \cos A$

$\angle A = 57°$

$\angle B = 49°$

$23^2 = 18^2 + 20^2 - 2 \cdot 18 \cdot 20 \cdot \cos C$

$529 = 724 - 720 \cos C$

$\frac{-195}{-220} = \frac{-720 \cos C}{-720}$

$.27083 = \cos C$

$\cos^{-1}) = 74°$

$\angle C = 74°$

Solve the system by substitution.

14) $x + y = 8$
$x - y = 6$

$y = 8 - x$

$x - (8 - x) = 6$

$x - 8 + x = 6$

$2x = 14$

$x = 7$

$y = 1$

Solve the system of equations by finding the reduced row echelon form for the augmented matrix.

15) $x - y + 4z = -3$
$3x + z = 0$
$x + 4y + z = 12$

on calculator

$$\begin{bmatrix} 1 & -1 & 4 & -3 \\ 3 & 0 & 1 & 0 \\ 1 & 4 & 1 & 12 \end{bmatrix}$$

rref $= \begin{bmatrix} 0 & 0 & 0 & \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(0, 3, 0)

Find the partial fraction decomposition.

16) $\frac{x+2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ $(x+1)(x-1)$

$\frac{Ax-A+Bx+B}{x^2-1} = \frac{(A+B)x-A+B}{x^2-1}$

$A+B=1$

$-A+B=2$

$2B=3$

$B=\frac{3}{2}$

$A=-\frac{1}{2}$

Solve the system graphically.

17) $y = x^2 + 4x - 2$

$y = 5x + 2$

$(2.561, 14.808)$ and $(-1.562, -5.808)$

Determine whether the matrices are inverses.

18) $\begin{bmatrix} 9 & 4 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} -0.2 & 0.2 \\ 0.2 & -0.45 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

no $A \cdot B \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find the matrix product, if possible.

19) $\begin{bmatrix} 1 & 3 & -3 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -3 & 1 \\ 0 & 4 \end{bmatrix}$

$r_1 \cdot c_1 = 3 - 9 + 0 = -6$

$r_1 c_2 = 0 + 3 - 12 = -9$

$r_2 \cdot c_1 = 9 + 0 + 0 = 9$

$r_2 \cdot c_2 = 0 + 0 + 16 = 16$

$\begin{bmatrix} -6 & -9 \\ 9 & 16 \end{bmatrix}$

Find a row echelon form or a reduced row echelon form, as indicated, for the given matrix.

20) Find a row echelon form for the matrix.

$\begin{bmatrix} 1 & -4 & -7 & 1 \\ -1 & 6 & -1 & 1 \\ -2 & 12 & 0 & 1 \end{bmatrix}$

$r_1 + r_2$ $\begin{bmatrix} 1 & -4 & -7 & 1 \\ 0 & 2 & -8 & 2 \\ -2 & 12 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -4 & -7 & 1 \\ 0 & 2 & -8 & 2 \\ 0 & 4 & -14 & 3 \end{bmatrix}$

$2r_1 + r_3$ $0 \quad -4 \quad 16 \quad -4$

$-2r_2 + r_3$

$\rightarrow \begin{bmatrix} 1 & -4 & -7 & 1 \\ 0 & 2 & -8 & 2 \\ 0 & 0 & 2 & -1 \end{bmatrix} \div 2 \div 2 \rightarrow \begin{bmatrix} 1 & -4 & -7 & 1 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} \end{bmatrix}$

Write the augmented matrix for the system.

21) $-2x + 7y + 5z = 66$
$7x + 7y + 6z = 99$
$2x + 4y - 2z = 18$

$$\begin{bmatrix} -2 & 7 & 5 & 66 \\ 7 & 7 & 6 & 99 \\ 2 & 4 & -2 & 18 \end{bmatrix}$$

Solve the problem.

22) How many committees of 5 people can be selected from 8 men and 8 women if the committee must have 3 men and 2 women?

$${}_8C_3 \cdot {}_8C_2 = 1568$$

Solve.

23) How many ways can a president, vice-president, and secretary be chosen from a club with 9 members?

$${}_9P_3 = 504$$

24) Write but do not evaluate the series defined by $\sum_{n=2}^{7} n(n+1)$ $= 6 + 12 + 20 + 30 + 42 + 56$

Write the series using summation notation.

25) $9 - 27 + 81 - 243 + \ldots$

$$\sum_{n=1}^{\infty} 9(-3)^{n-1}$$

26) $11 + 13 + 15 + 17 + \ldots + 2n+1 \ldots$

$$\sum_{n=5}^{\infty} 2n+1$$

Solve the problem.

27) Suppose you receive a shipment of 33 computers, two of which are defective. What is the probability of picking a sample of 6 computers with exactly one bad computer in it? (Round approximations to the nearest thousandth.)

$p = \frac{2}{33}$ $q = \frac{31}{33}$

$$\binom{6}{1}\left(\frac{2}{33}\right)^1\left(\frac{31}{33}\right)^5 = .266$$

28) A box contains 4 slips of paper, on each of which is written the number 1, 2, 3, or 4, respectively. A slip is drawn and the number on it is noted. That slip is put aside, another drawn, and the number or it noted. What is the probability that
a) the sum of the two numbers is 5?
b) the first number drawn is a 2 and the second number is not 5?
c) the sum of the two numbers is not 10?

Find the derivative of the function at the specified point.

29) $f(x) = -4x^2 + 12x$ at $x = 7$

$-8x+12 \rightarrow -8\cdot 7+12 = -44$

Derivative = slope of the tangent line

Find the equation of the tangent line to the curve when x has the given value.

30) $f(x) = 5x^2 + x$; $x = -4$ $f(-4) = 76$

$y-76 = -39(x+4)$

$10x+1$

$10(-4)+1$

$-40+1 = -39$

Find the derivative of the function using the definition of derivative.

31) $f(x) = 3x^2 - 4x$

$6x-4$

Find the limit of the function by using direct substitution.

32) $\lim_{x\to 2} (x^2 + 8x - 2)$ $= 2^2+8\cdot 2-2 = 18$

33) $\lim_{x\to 0} (\sqrt{x} - 2)$ $= \sqrt{0}-2 = -2$

Find the limit of the function algebraically.

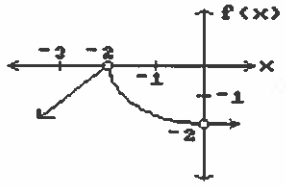
34) $\lim_{x\to 0} \frac{x^3 + 12x^2 - 5x}{5x}$ $\lim_{x\to 0} \frac{x^2+12x-5}{5} = \frac{-5}{5} = -1$

35) $\lim_{x \to 6} \frac{x+6}{(x-6)^2}$ = NO Limit

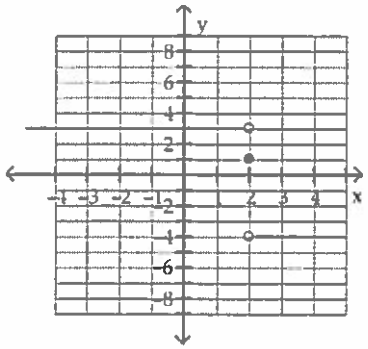
36) $\lim_{x \to 8} \frac{x^2-64}{x-8}$ $= \frac{(x-8)(x+8)}{(x-8)} \rightarrow 16$

Use the given graph to determine the limit, if it exists.

37) $\lim_{x \to 0} f(x)$ $= -2$



38)



Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.

-4 $\quad$ 3

Use graphs and tables to find the limit and identify any vertical asymptotes.

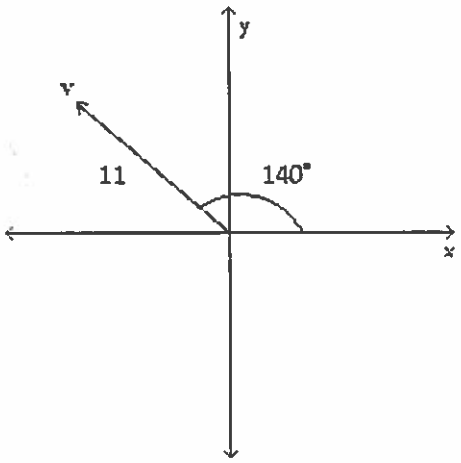
39) $\lim_{x \to -9} \frac{1}{x^2-81}$

Vertical asymptotes $x=-9$ $x=9$

$\lim_{x \to -9} \frac{1}{x^2-81}$ doesn't exist

Find the component form of the vector v.

40)

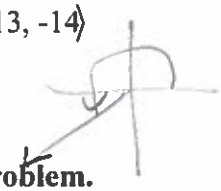


$\langle 11\cos 140, 11\sin 140\rangle$

$\langle -8.426, 7.071\rangle$

Find the magnitude and direction angle for the following vector. Give the direction angle as an angle in [0°, 360°) rounded to the nearest tenth.

41) $\langle -13, -14\rangle$



$\theta = \tan^{-1}\left(\frac{-14}{-13}\right) = 47 + 180 = 227^\circ$

magnitude $= \sqrt{(-13)^2 + (-14)^2} \approx 19.105$

Solve the problem.

42) A force of 90 lb acts on an object at an angle of 55°. A second force of 100 lb acts on the object at an angle of -60°. Find the direction and magnitude of the resultant force.

$\langle 90\cos 55 + 100\cos -60, 90\sin 55 + 100\sin -60\rangle = \langle 101.622, -12.879\rangle$

$\sqrt{101.622^2 + (-12.879)^2} = 102.435$ lbs

$\theta = \tan^{-1}\left(\frac{-12.879}{101.622}\right) = -7^\circ$

Find the angle between the given vectors to the nearest tenth of a degree.

43) $\mathbf{u} = \langle 1, 1\rangle$, $\mathbf{v} = \langle 7, -9\rangle$ $\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \frac{-2}{\sqrt{50}\cdot\sqrt{130}} \rightarrow 91^\circ$

Eliminate the parameter.

44) $x = \sqrt{t}$, $y = 2t + 5$

$x^2 = t$ $y = 2(x^2) + 5$

45) $x = t + 4$, $y = t^2$

$x = t + 4$

$t = x - 4$

$y = (x-4)^2$ or $x^2 - 8x + 16$

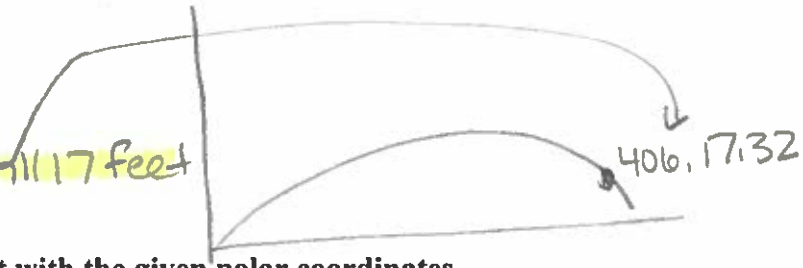
Solve the problem using a graphing calculator.

46) Determine whether a baseball hit 118 feet per second at an angle of 40° relative to level ground will clear a 10-foot wall 400 feet away.

$x = 118\cos 40° \cdot t$

$y = -16t^2 + 118\sin 40° \cdot t$

yes the baseball is 10 feet high 406 feet away



Find the rectangular coordinates of the point with the given polar coordinates.

47) (8, 240°)

$x = r\cos\theta = 8\cos 240 = -4$

$y = r\sin\theta = 8 \cdot \sin 240 = 5.142$

$(-4, 5.142)$

Determine two pairs of polar coordinates for the point with $0° \le \theta < 360°$.

48) (4, 4)

$r = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{4}{4}\right) = 45°$

$(4\sqrt{2}, 45°)$

$(-4\sqrt{2}, 225°)$

Find an equivalent equation in rectangular coordinates.

49) $r = 2(\sin\theta - \cos\theta)$

$r = 2\sin\theta - 2\cos\theta$

$r \cdot r = 2r\sin\theta - 2r\cos\theta$

$r^2 = 2r\sin\theta - 2r\cos\theta$

$x^2 + y^2 = 2x - 2y$

Solve the problem.

50) What is the maximum r-value for the polar equation $r = 3\cos 2\theta$? Note that a maximum r-value is a maximum value of $|r|$, the maximum distance from the pole. Analyze the function:

r-max is 3 Rose curve

Domain: All reals

Range: [-3, 3]

continuous

x, y-axis, origin symmetry

Bounded

4 petals

no asymptotes

51) What is the maximum r-value for the polar equation $r = -3 + 5 \sin\theta$? Note that a maximum r-value is a maximum value of $|r|$, the maximum distance from the pole. Analyze the function

Inner loop limacon r-max = 2

Domain All reals y-axis symmetry

Range [-8, 2] Bounded

Continuous No Asymptotes

Find an equivalent equation in polar coordinates.

52) $y = 11$

$r\sin\theta = 11$

$r = 11/\sin\theta$

$r = 11\csc\theta$

Horizontal Line

Solve the problem.

53) Find the work done by a force F of 21 pounds acting in the direction $\langle 2, 5 \rangle$ in moving an object 7 feet from (0, 0) to (7, 0).

$W = F \cdot \cos\theta \cdot d$

$W = 21 \cdot \cos 68.2^\circ \cdot 7 = 54.594$ ft-lbs