6. (a) $\cos A = \dfrac{y^2 - x^2 - 25}{-10} = \dfrac{x^2 - y^2 + 25}{10}$

(b) $A = \cos^{-1}\left(\dfrac{x^2 - y^2 + 25}{10}\right)$

7. One answer: $(x - 1)(x - 2) = x^2 - 3x + 2$.
Generally: $(x - a)(x - b) = x^2 - (a + b)x + ab$ for any two positive numbers a and b.

8. One answer: $(x - 1)(x + 1) = x^2 - 1$.
Generally, $(x - a)(x + b) = x^2 - (a - b)x - ab$ for any two positive numbers a and b.

9. One answer: $(x - i)(x + i) = x^2 + 1$

10. One answer: $(x - 1)^2 = x^2 - 2x + 1$.
Generally: $(x - a)^2 = x^2 - 2ax + a^2$ for any positive number a.

Section 5.6 Exercises

1. Given: $B = 131°, c = 8, a = 13$ — an SAS case.
$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{369.460} \approx 19.2$;
$C = \cos^{-1}\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.949) \approx 18.3°$;
$A = 180° - (B + C) \approx 30.7°$.

2. Given: $C = 42°, b = 12, a = 14$ — an SAS case.
$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx \sqrt{90.303} \approx 9.5$;
$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.167) \approx 80.3°$;
$B = 180° - (A + C) \approx 57.7°$.

3. Given: $a = 27, b = 19, c = 24$ — an SSS case.
$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.228) \approx 76.8°$;
$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.728) \approx 43.2°$;
$C = 180° - (A + B) \approx 60°$.

4. Given: $a = 28, b = 35, c = 17$ — an SSS case.
$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.613) \approx 52.2°$;
$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.159) \approx 99.2°$;
$C = 180° - (A + B) \approx 28.6°$.

5. Given: $A = 55°, b = 12, c = 7$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{96.639} \approx 9.8$;
$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.011) \approx 89.3°$;
$C = 180° - (A + B) \approx 35.7°$.

6. Given: $B = 35°, a = 43, c = 19$ — an SAS case.
$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{871.505} \approx 29.5$;
$C = \cos^{-1}\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.929) \approx 21.7°$;
$A = 180° - (B + C) \approx 123.3°$.

7. Given: $a = 12, b = 21, C = 95°$ — an SAS case.
$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx \sqrt{628.926} \approx 25.1$;
$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.879) \approx 28.5°$;
$B = 180° - (A + C) \approx 56.5°$.

8. Given: $b = 22, c = 31, A = 82°$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{1255.167} \approx 35.4$;
$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.788) \approx 37.9°$;
$C = 180° - (A + B) \approx 60.1°$.

9. No triangles possible $(a + c = b)$

10. No triangles possible $(a + b < c)$

11. Given: $a = 3.2, b = 7.6, c = 6.4$ — an SSS case.
$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.909) \approx 24.6°$;
$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.160) \approx 99.2°$;
$C = 180° - (A + B) \approx 56.2°$.

12. No triangles possible $(a + b < c)$

Exercises 13–16 are SSA cases, and can be solved with either the law of sines or the law of cosines. The law of cosines solution is shown.

13. Given: $A = 42°, a = 7, b = 10$ — an SSA case. Solve the quadratic equation $7^2 = 10^2 + c^2 - 2(10)c\cos 42°$, or $c^2 - (14.862\ldots)c + 51 = 0$; there are two positive solutions: ≈ 9.487 *or* 5.376. Since $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$:
$c_1 \approx 9.487$, $B_1 \approx \cos^{-1}(0.294) \approx 72.9°$, and
$C_1 = 180° - (A + B_1) \approx 65.1°$,
or
$c_2 \approx 5.376$, $B_2 \approx \cos^{-1}(-0.294) \approx 107.1°$, and
$C_2 = 180° - (A + B_2) \approx 30.9°$.

14. Given: $A = 57°, a = 11, b = 10$ — an SSA case. Solve the quadratic equation $11^2 = 10^2 + c^2 - 2(10)C\cos 57°$, or $c^2 - (10.893)c - 21 = 0$; there is one positive solution $c = 12.564$. Since $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$,
$B \approx \cos^{-1}(0.647) \approx 49.7°$ and $C = 180° - (A + B) = 73.3°$.

15. Given: $A = 63°, a = 8.6, b = 11.1$ — an SSA case. Solve the quadratic equation
$8.6^2 = 11.1^2 + c^2 - 2(11.1)c\cos 63°$, or
$c^2 - (10.079)c + 49.25 = 0$; there are no real solutions, so there is no triangle.

16. Given: $A = 71°, a = 9.3, b = 8.5$ — an SSA case. Solve the quadratic equation
$9.3^2 = 8.5^2 + c^2 - 2(8.5)c\cos 71°$, or
$c^2 - (5.535)c - 14.24 = 0$; there is one positive solution: $c \approx 7.447$. Since $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$,
$B \approx \cos^{-1}(0.503) \approx 59.8°$ and $C = 180° - (A + B) \approx 49.2°$.

17. Given: $A = 47°, b = 32, c = 19$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{555.689} \approx 23.573$,
so Area $\approx \sqrt{49431.307} \approx 222.33$ ft^2 (using Heron's formula). Or, use $A = \dfrac{1}{2}bc\sin A$.

18. Given: $A = 52°, b = 14, c = 21$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{274.991} \approx 16.583$,
so Area $\approx \sqrt{13418.345} \approx 115.84$ m^2 (using Heron's formula). Or, use $A = \dfrac{1}{2}bc\sin A$.