(e) The first regression indicates that the data are periodic and nearly sinusoidal. The second regression indicates that the *variation* of the data around the predicted values is also periodic and nearly sinusoidal. Periodic variation around periodic models is a predictable consequence of bodies orbiting bodies, but ancient astronomers had a difficult time reconciling the data with their simpler models of the universe.

Section 5.5 The Law of Sines

Exploration 1

1. If $BC \le AB$, the segment will not reach from point B to the dotted line. On the other hand, if $BC > AB$, then a circle of radius BC will intersect the dotted line in a unique point. (Note that the line only extends to the left of point A.)
2. A circle of radius BC will be tangent to the dotted line at C if $BC = h$, thus determining a unique triangle. It will miss the dotted line entirely if $BC < h$, thus determining zero triangles.
3. The second point (C_2) is the reflection of the first point (C_1) on the other side of the altitude.
4. $\sin C_2 = \sin(\pi - C_1) = \sin \pi \cos C_1 - \cos \pi \sin C_1 = \sin C_1$.
5. If $BC \ge AB$, then BC can only extend to the right of the altitude, thus determining a unique triangle.

Quick Review 5.5

1. $a = bc/d$
2. $b = ad/c$
3. $c = ad/b$
4. $d = bc/a$
5. $\dfrac{7 \sin 48°}{\sin 23°} \approx 13.314$
6. $\dfrac{9 \sin 121°}{\sin 14°} \approx 31.888$
7. $x = \sin^{-1} 0.3 \approx 17.458$
8. $x = 180° - \sin^{-1} 0.3 \approx 162.542$
9. $x = 180° - \sin^{-1}(-0.7) \approx 224.427$
10. $x = 360° + \sin^{-1}(-0.7) \approx 315.573$

Section 5.5 Exercises

1. Given: $b = 3.7, B = 45°, A = 60°$ — an AAS case.
$C = 180° - (A + B) = 75°$;
$a = \dfrac{b \sin A}{\sin B} = \dfrac{3.7 \sin 60°}{\sin 45°} \approx 4.5$;
$c = \dfrac{b \sin C}{\sin B} = \dfrac{3.7 \sin 75°}{\sin 45°} \approx 5.1$

2. Given: $c = 17, B = 15°, C = 120°$ — an AAS case.
$A = 180° - (B + C) = 45°$;
$a = \dfrac{c \sin A}{\sin C} = \dfrac{17 \sin 45°}{\sin 120°} \approx 13.9$;
$b = \dfrac{c \sin B}{\sin C} = \dfrac{17 \sin 15°}{\sin 120°} \approx 5.1$

3. Given: $A = 100°, C = 35°, a = 22$ — an AAS case.
$B = 180° - (A + C) = 45°$;
$b = \dfrac{a \sin B}{\sin A} = \dfrac{22 \sin 45°}{\sin 100°} \approx 15.8$;
$c = \dfrac{a \sin C}{\sin A} = \dfrac{22 \sin 35°}{\sin 100°} \approx 12.8$

4. Given: $A = 81°, B = 40°, b = 92$ — an AAS case.
$C = 180° - (A + B) = 59°$;
$a = \dfrac{b \sin A}{\sin B} = \dfrac{92 \sin 81°}{\sin 40°} \approx 141.4$;
$c = \dfrac{b \sin C}{\sin B} = \dfrac{92 \sin 59°}{\sin 40°} \approx 122.7$

5. Given: $A = 40°, B = 30°, b = 10$ — an AAS case.
$C = 180° - (A + B) = 110°$;
$a = \dfrac{b \sin A}{\sin B} = \dfrac{10 \sin 40°}{\sin 30°} \approx 12.9$;
$c = \dfrac{b \sin C}{\sin B} = \dfrac{10 \sin 110°}{\sin 30°} \approx 18.8$

6. Given: $A = 50°, B = 62°, a = 4$ — an AAS case.
$C = 180° - (A + B) = 68°$;
$b = \dfrac{a \sin B}{\sin A} = \dfrac{4 \sin 62°}{\sin 50°} \approx 4.6$;
$c = \dfrac{a \sin C}{\sin A} = \dfrac{4 \sin 68°}{\sin 50°} \approx 4.8$

7. Given: $A = 33°, B = 70°, b = 7$ — an AAS case.
$C = 180° - (A + B) = 77°$;
$a = \dfrac{b \sin A}{\sin B} = \dfrac{7 \sin 33°}{\sin 70°} \approx 4.1$;
$c = \dfrac{b \sin C}{\sin B} = \dfrac{7 \sin 77°}{\sin 70°} \approx 7.3$

8. Given: $B = 16°, C = 103°, c = 12$ — an AAS case.
$A = 180° - (B + C) = 61°$;
$a = \dfrac{c \sin A}{\sin C} = \dfrac{12 \sin 61°}{\sin 103°} \approx 10.8$;
$b = \dfrac{c \sin B}{\sin C} = \dfrac{12 \sin 16°}{\sin 103°} \approx 3.4$

9. Given: $A = 32°, a = 17, b = 11$ — an SSA case.
$h = b \sin A \approx 5.8$; $h < b < a$, so there is one triangle.
$B = \sin^{-1}\left(\dfrac{b \sin A}{a}\right) = \sin^{-1}(0.342\ldots) \approx 20.1°$
$C = 180° - (A + B) \approx 127.9°$;
$c = \dfrac{a \sin C}{\sin A} = \dfrac{17 \sin 127.9°}{\sin 32°} \approx 25.3$

10. Given: $A = 49°, a = 32, b = 28$ — an SSA case.
$h = b \sin A \approx 21.1$; $h < b < a$, so there is one triangle.
$B = \sin^{-1}\left(\dfrac{b \sin A}{a}\right) = \sin^{-1}(0.660\ldots) \approx 41.3°$
$C = 180° - (A + B) = 89.7°$;
$c = \dfrac{a \sin C}{\sin A} = \dfrac{32 \sin 89.7°}{\sin 49°} \approx 42.4$

11. Given: $B = 70°, b = 14, c = 9$ — an SSA case.
$h = c \sin B \approx 8.5$; $h < c < b$, so there is one triangle.
$C = \sin^{-1}\left(\dfrac{c \sin B}{b}\right) = \sin^{-1}(0.604\ldots) \approx 37.2°$
$A = 180° - (B + C) \approx 72.8°$;
$a = \dfrac{b \sin A}{\sin B} = \dfrac{14 \sin 72.8°}{\sin 70°} \approx 14.2$