12. Given: $C = 103°, b = 46, c = 61$ — an SSA case. $h = b \sin C \approx 44.8$; $h < b < c$, so there is one triangle.

$$B = \sin^{-1}\left(\frac{b \sin C}{c}\right) = \sin^{-1}(0.734\ldots) \approx 47.3°$$

$$A = 180° - (B + C) = 29.7°;$$

$$a = \frac{c \sin A}{\sin C} = \frac{61 \sin 29.7°}{\sin 103°} \approx 31.0$$

13. Given: $A = 36°, a = 2, b = 7$. $h = b \sin A \approx 4.1$; $a < h$, so no triangle is formed.

14. Given: $B = 82°, b = 17, c = 15$. $h = c \sin B \approx 14.9$; $h < c < b$, so there is one triangle.

15. Given: $C = 36°, a = 17, c = 16$. $h = a \sin C \approx 10.0$; $h < c < a$, so there are two triangles.

16. Given: $A = 73°, a = 24, b = 28$. $h = b \sin A \approx 26.8$; $a < h$, so no triangle is formed.

17. Given: $C = 30°, a = 18, c = 9$. $h = a \sin C \approx 9$; $h = c$, so there is one triangle.

18. Given: $B = 88°, b = 14, c = 62$. $h = c \sin B \approx 62.0$; $b < h$, so no triangle is formed.

19. Given: $A = 64°, a = 16, b = 17$. $h = b \sin A \approx 15.3$; $h < a < b$, so there are two triangles.

$$B_1 = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.954\ldots) \approx 72.7°$$

$$C_1 = 180° - (A + B_1) \approx 43.3°;$$

$$c_1 = \frac{a \sin C_1}{\sin A} = \frac{16 \sin 43.3°}{\sin 64°} \approx 12.2$$

Or (with B obtuse):

$$B_2 = 180° - B_1 \approx 107.3°;$$

$$C_2 = 180° - (A + B_2) \approx 8.7°;$$

$$c_2 = \frac{a \sin C_2}{\sin A} \approx 2.7$$

20. Given: $B = 38°, b = 21, c = 25$. $h = c \sin B \approx 15.4$; $h < b < c$, so there are two triangles.

$$C_1 = \sin^{-1}\left(\frac{c \sin B}{b}\right) = \sin^{-1}(0.732\ldots) \approx 47.1°$$

$$A_1 = 180° - (B + C_1) \approx 94.9°;$$

$$a_1 = \frac{b \sin A_1}{\sin B} = \frac{21 \sin 94.9°}{\sin 38°} \approx 34.0$$

Or (with C obtuse):

$$C_2 = 180° - C_1 \approx 132.9°;$$

$$A_2 = 180° - (B + C_2) \approx 9.1°;$$

$$a_2 = \frac{b \sin A_2}{\sin B} \approx 5.4$$

21. Given: $C = 68°, a = 19, c = 18$. $h = a \sin C \approx 17.6$; $h < c < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin C}{c}\right) = \sin^{-1}(0.978\ldots) \approx 78.2°$$

$$B_1 = 180° - (A + C) \approx 33.8°;$$

$$b_1 = \frac{c \sin B_1}{\sin C} = \frac{18 \sin 33.8°}{\sin 68°} \approx 10.8$$

Or (with A obtuse):

$$A_2 = 180° - A_1 \approx 101.8°;$$

$$B_2 = 180° - (A_2 + C) \approx 10.2°;$$

$$b_2 = \frac{c \sin B_2}{\sin C} \approx 3.4$$

22. Given: $B = 57°, a = 11, b = 10$. $h = a \sin B \approx 9.2$; $h < b < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.922\ldots) \approx 67.3°$$

$$C_1 = 180° - (A_1 + B) \approx 55.7°;$$

$$c_1 = \frac{b \sin C_1}{\sin B} = \frac{10 \sin 55.7°}{\sin 57°} \approx 9.9$$

Or (with A obtuse):

$$A_2 = 180° - A_1 \approx 112.7°;$$

$$C_2 = 180° - (A_2 + B) \approx 10.3°;$$

$$c_2 = \frac{b \sin C_2}{\sin B} \approx 2.1$$

23. $h = 10 \sin 42° \approx 6.691$, so:

(a) $6.691 < b < 10$.

(b) $b \approx 6.691$ or $b \geq 10$.

(c) $b < 6.691$

24. $h = 12 \sin 53° \approx 9.584$, so:

(a) $9.584 < c < 12$.

(b) $c \approx 9.584$ or $c \geq 12$.

(c) $c < 9.584$

25. **(a)** No: this is an SAS case

(b) No: only two pieces of information given.

26. **(a)** Yes: this is an AAS case.

$$B = 180° - (A + C) = 32°;$$

$$b = \frac{a \sin B}{\sin A} = \frac{81° \sin 32°}{\sin 29°} \approx 88.5;$$

$$c = \frac{a \sin C}{\sin A} = \frac{81 \sin 119°}{\sin 29°} \approx 146.1$$

(b) No: this is an SAS case.

27. Given: $A = 61°, a = 8, b = 21$ — an SSA case. $h = b \sin A = 18.4$; $a < h$, so no triangle is formed.

28. Given: $B = 47°, a = 8, b = 21$ — an SSA case. $h = a \sin B \approx 5.9$; $h < a < b$, so there is one triangle.

$$A = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.278\ldots) \approx 16.2°$$

$$C = 180° - (A + B) = 116.8°;$$

$$c = \frac{b \sin C}{\sin B} = \frac{21 \sin 116.8°}{\sin 47°} \approx 25.6$$

29. Given: $A = 136°, a = 15, b = 28$ — an SSA case. $h = b \sin A \approx 19.5$; $a < h$, so no triangle is formed.

30. Given: $C = 115°, b = 12, c = 7$ — an SSA case. $h = b \sin C \approx 10.9$; $c < h$, so no triangle is formed.

31. Given: $B = 42°, c = 18, C = 39°$ — an AAS case.

$$A = 180° - (B + C) = 99°;$$

$$a = \frac{c \sin A}{\sin C} = \frac{18 \sin 99°}{\sin 39°} \approx 28.3;$$

$$b = \frac{c \sin B}{\sin C} = \frac{18 \sin 42°}{\sin 39°} \approx 19.1$$

32. Given: $A = 19°, b = 22, B = 47°$ — an AAS case.

$$C = 180° - (A + B) = 114°;$$

$$a = \frac{b \sin A}{\sin B} = \frac{22 \sin 19°}{\sin 47°} \approx 9.8;$$

$$c = \frac{b \sin C}{\sin B} = \frac{22 \sin 114°}{\sin 47°} \approx 27.5$$