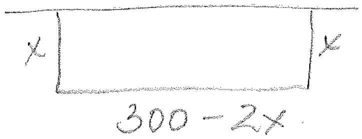
Applications to Derivatives   Optimization Problems   Name________________________

1. A rancher has 300 feet of fencing to enclose a pasture bordered on one side by a river. The river side of the pasture needs no fence. Find the dimensions of the pasture that will produce a pasture with a maximum area.



$$\text{Area} = x(300 - 2x)$$

$$A(x) = 300x - 2x^2$$

$$A'(x) = 300 - 4x$$

$$0 = 300 - 4x$$

$$-300 = -4x$$

$$75 = x$$

$A''(x) = -4 < 0 \Rightarrow$ concave down which justifies it is a max.

$\boxed{\text{Dimensions are } 75 \times 150}$

2. A manufacturer determines that x employees on a certain production line will produce y units per month where $y = 75x^2 - 0.2x^4$. To obtain maximum monthly production, how many employees should be assigned to the production line?

Maximize production $y = 75x^2 - 0.2x^4$

$$y' = 150x - 0.8x^3$$

$$0 = x(150 - 0.8x^2)$$

$$0 \qquad \frac{-150}{-0.8} = \frac{-0.8x^2}{-0.8}$$

$$187.5 = x^2$$

$$\pm 13.69 = x$$

$$14 \approx x$$

employees.

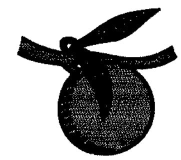
$$y''(x) = 150 - 2.4x^2$$

At $x = 14$, 2nd der. is negative $\Rightarrow$ concave down meaning it is a max.

3. There are 50 orange trees in an orchard. Each tree (on average) produces 800 oranges. Due to soil and irrigation conditions, for each additional tree planted in the orchard, the output per tree for all trees drops by 10 oranges. How many trees should be added to the existing orchard in order to maximize the total output of trees?
(Hint: output equals number of trees times number of oranges per tree)

Out = (# of trees)(# of oranges/tree)

Let x = # of trees to be added



Max Out = $(50+x)(800-10x)$

Output = $40{,}000 + 300x - 10x^2$

Der(Out) = $300 - 20x$

$P''(x) = -20 < 0 \Rightarrow$ maximum concave down.

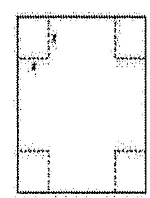
$0 = 300 - 20x$

$-300 = -20x$

$15 = x$

$\boxed{\text{\# of trees is 15.}}$

4. An open box is to be constructed from cardboard by cutting out squares of equal size in the corners and then folding up the sides. If the cardboard is 6 inches by 11 inches, determine the volume of the largest box which can be so constructed.



$V = lwh$

$V = (6-2x)(11-2x)x$

$V = 66x - 34x^2 + 4x^3$

$V'(x) = 66 - 68x + 12x^2$

Calc needed

$V''(x) = -68 + 24x$

$= -68 + 24(1.24)$

$= -38.24 < 0 \Rightarrow$ max.

$x = 1.24$ or $\cancel{4.42}$ not in domain.

$\boxed{V = 37.152 \text{ in}^3}$

5. If the price charged for a candy bar is $p(x)$ cents, then x thousand candy bars will be sold in a certain city, where $p(x)=125-\frac{x}{14}$. How many candy bars must be sold to maximize revenue?

Note: Revenue = price times quantity where quantity is x.

$p(x) = 125 - \frac{x}{14}$

$Rev = x\left(125 - \frac{x}{14}\right)$

$= 125x - \frac{x^2}{14}$

$R'(x) = 125 - \frac{1}{7}x$

$0 = 125 - \frac{1}{7}x$

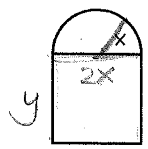
$-125 = -\frac{1}{7}x$

$875 = x$

$R''(x) = -\frac{1}{7} < 0 \Rightarrow$ max.

$\boxed{875{,}000 \text{ candy bars}}$

6. A window is in the form of a rectangle surmounted by a semicircle. This is called a Norman window. The total perimeter is 30 feet. Find the proportions of the window that will admit the most light (greatest area.) Neglect the thickness of the frame. Hint: let the radius $=x$ and then w, *the horizontal width of the window* $=2x$.



$P = 2y + 2x + \pi x$

$30 = 2y + 2x + \pi x$

$30 - 2x - \pi x = 2y$

$\boxed{15 - x - \frac{\pi x}{2} = y}$

$A = 2x\left(15 - x - \frac{\pi x}{2}\right) + \frac{1}{2}\pi x^2$

$A = 30x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

$A'(x) = 30 - 4x - 2\pi x + \pi x$

$0 = 30 - 4x - \pi x$

$-30 = -4x - \pi x$

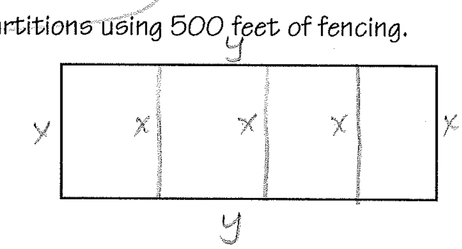
$-30 = x(-4 - \pi)$

$\frac{30}{4+\pi} = x$

$x \approx 4.2$
$y \approx 4.2$

Dimensions
$w \approx 4.2(2) = 8.4$
$l = 4.2$

7. You are to build a rectangular garden with three parallel interior partitions using 500 feet of fencing. What dimensions will maximize the total area of the garden?



$P = 5x + 2y$

$500 = 5x + 2y$

$\frac{500 - 5x}{2} = y$

$\boxed{250 - \frac{5x}{2} = y}$

$A = lw$

$= x\left(250 - \frac{5}{2}x\right)$

$A(x) = 250x - \frac{5}{2}x^2$

$A'(x) = 250 - 5x$

$0 = 250 - 5x$

$\frac{-250}{-5} = x$

$50 = x$
$125 = y$

8. Find two nonnegative, nonzero numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

$1^{st}\# = x$

$2^{nd}\# = y$

$x + y = 9$

$x = 9 - y$

$N = (9-y)(y)^2$

$N = 9y^2 - y^3$

$N'(y) = 18y - 3y^2$

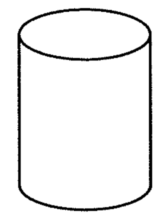
$0 = y(18 - 3y)$

$-18 = -3y$

$6 = y$

$\boxed{y = 6,\ x = 3}$

9. A container in the shape of a right circular cylinder with no top has surface area 3π ft.^{2} What height, h, and base radius, r, will maximize the volume of the cylinder ?



$V = \pi r^2 h$

$SA = 3\pi$

$SA = \pi r^2 + 2\pi r h$

$3\pi = \pi r^2 + 2\pi r h$

$\dfrac{3\pi - \pi r^2}{2\pi r} = h$

$\boxed{h = \dfrac{3 - r^2}{2r}}$

$V = \pi r^2 \left(\dfrac{3 - r^2}{2r}\right)$

$V = \pi r \left(\dfrac{3 - r^2}{2}\right)$

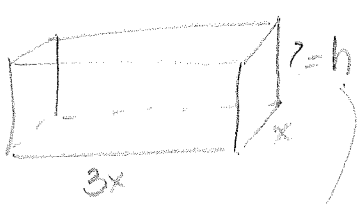
$V = \dfrac{\pi}{2}(3r - r^3)$

$V'(r) = \dfrac{\pi}{2}(3 - 3r^2)$

$0 = \dfrac{\pi}{2}(3 - 3r^2)$ ✓

$r = 1,\ h = 1$?

10. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50ft^3 determine the dimensions that will minimize the cost to build the box.



$V = lwh$

$\dfrac{V}{lw} = h$

$\boxed{\dfrac{50}{3x^2} = h}$

top/bottom sides

$SA = 2(3x^2) + 2(3x)\left(\dfrac{50}{3x^2}\right) + 2(x)\left(\dfrac{50}{3x^2}\right)$

$Cost = 10(2)(3x^2) + 6(2)(3x)\left(\dfrac{50}{3x^2}\right) + 6(2)(x)\dfrac{50}{3x^2}$

$Cost = 60x^2 + \dfrac{600}{x} + \dfrac{200}{x}$

$Cost = 60x^2 + \dfrac{800}{x}$

$C'(x) = 120x - \dfrac{800}{x^2}$

$0 = 120x - \dfrac{800}{x^2}$

$1.88 = x$

Check $C''(x)$ to find if $> 0 \Rightarrow$ minimum

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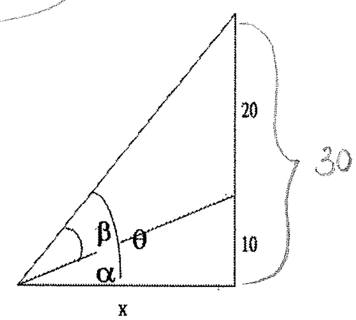
11. A movie screen on a wall is 20 feet high and 10 feet above the floor. At what distance x from the front of the room should you position yourself so that the viewing angle β of the movie screen is as large as possible? (See diagram.)

Maximize β, $\beta = \theta - \alpha$



$$\tan\theta = \frac{30}{x} \qquad \tan\alpha = \frac{10}{x}$$

$$\theta = \tan^{-1}\left(\frac{30}{x}\right) \qquad \alpha = \tan^{-1}\left(\frac{10}{x}\right)$$

$$\therefore \beta = \tan^{-1}\left(\frac{30}{x}\right) - \tan^{-1}\left(\frac{10}{x}\right)$$

$$\frac{d\beta}{dx} = \frac{-\frac{30}{x^2}}{1+\frac{900}{x^2}} - \frac{-\frac{10}{x^2}}{1+\frac{100}{x^2}}$$ LCD. simplify fractions.

$$= \frac{-30}{x^2+900} + \frac{10}{x^2+100}$$

$$= \frac{-30x^2 - 3000 + 10x^2 + 9000}{(x^2+900)(x^2+100)}$$ Algebra

$$= \frac{-20x^2+6000}{(x^2+900)(x^2+100)}$$

$$0 = -20x^2 + 6000$$

$$\frac{-6000}{-20} = \frac{-20x^2}{-20}$$

$$\sqrt{300} = x^2$$

$$\approx 17.3\text{ ft} = x$$