Precalculus Key

C.7 Review 3-4

Name

DIRECTIONS: SHOW ALL WORK AND SET UPS FOR FULL CREDIT!

Use Gaussian elimination to solve the system of equations.

1) Show all work.

$$\begin{cases} x + y + z = -6 \\ x - y + 3z = -10 \\ 3x + y + z = -2 \end{cases}$$

$-1 \cdot R_1 + R_2 \to R_2$

$-x - y - z = 6$

$x - y + 3z = -10$

$-2y + 2z = -4 \div -2 = y - 2z = 2$

$$\begin{cases} x+y+z=-6 \\ y+-2z=2 \\ 3x+y+z=-2 \end{cases}$$

$-3 \cdot R_1 + R_3 \to R_3$

$$\begin{cases} -3x-3y-3z=18 \\ 3x+y+z=-2 \end{cases}$$

$-2y-2z=16 \div -2 = y+z=-8$

$$\begin{cases} x+y+z=-6 \\ y-2z=2 \\ y+z=-8 \end{cases}$$

$-1 \cdot R_2 + R_3$

$-y+z=-2$

$y+z=-8$

$2z=-10$

$\boxed{z=-5}$

$y+z=-8$

$y-5=-8$

$\boxed{y=-3}$

$x+-3+-5=-6$

$x-8=-6$

$+8 \quad +8$

$\boxed{x=2}$

$(2,-3,-5)$

Solve the system of equations by finding the reduced row echelon form for the augmented matrix.

2) Use Calculator show augmented matrix.

$$\begin{aligned} x + y + z &= -10 \\ x - y + 3z &= -8 \\ 5x + y + z &= -14 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & -10 \\ 1 & -1 & 3 & -8 \\ 5 & 1 & 1 & -14 \end{bmatrix} \quad \text{rref} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$x=-1$

$y=-5$

$z=-4$

Write the terms for the partial fraction decomposition of the rational function. Do not solve for the constants.

3) $\dfrac{x + 2}{x(x^2 + 5x - 6)} = \dfrac{A}{x} + \dfrac{B}{x^2+5x-6}$

Find a row echelon form or a reduced row echelon form, as indicated, for the given matrix.

4) Find a row echelon form for the matrix. No calculator

$$\begin{bmatrix} 1 & -4 & 1 & 2 \\ -1 & 6 & -8 & -6 \\ -2 & 12 & -14 & 4 \end{bmatrix} \xrightarrow{r_1+r_2 \to r_2} \begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 2 & -7 & -4 \\ -2 & 12 & -14 & 4 \end{bmatrix} \xrightarrow{2 \cdot r_1 + r_3} \begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 2 & -7 & -4 \\ 0 & 4 & -12 & 8 \end{bmatrix}$$

$$\xrightarrow{-2r_2+r_3} \begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 2 & -7 & -4 \\ 0 & 0 & 2 & 16 \end{bmatrix} \quad \begin{bmatrix} 1 & -4 & 1 & 2 \\ 0 & 1 & -3.5 & -2 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

Find the partial fraction decomposition.

5) $\frac{3x - 1}{x(x + 1)} = \frac{A}{x} + \frac{B}{x + 1}$

$$-\frac{1}{x} + \frac{4}{x+1}$$

$$\frac{A(x+1)}{x\,(x+1)} + \frac{B\,(x)}{x+1\,(x)} = \frac{Ax+A+Bx}{(x+1)x}$$

$$\frac{(A+B)x+A}{x(x+1)}$$

$A+B=3$

$A=-1$ so $B=4$

6) $\frac{5x - 2}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x + 2} + \frac{C}{x - 2}$

$$\frac{A}{x}\frac{(x+2)(x-2)}{(x+2)(x-2)} + \frac{B\,(x-2)\cdot x}{(x+2)(x-2)\cdot x} + \frac{C\,(x+2)\cdot x}{(x-2)\,(x+2)\cdot x}$$

$$= Ax^2-4A+Bx^2+-2Bx+Cx^2+2Cx$$

$$=(A+B+C)x^2+(-2B+2C)x-4A$$

$A+B+C=0$

$-2B+5C=5$

$-4A=-2$

Answer the question.

7) Find a, b, and c so that the graph of the equation $y = ax^2 + bx + c$ passes through the points (5, 97), (3, 41), and (2, 22).

$25a+5b+c=97$

$9a+3b+c=41$

$4a+2b+c=22$

$$\begin{bmatrix} 25 & 5 & 1 & 97 \\ 9 & 3 & 1 & 41 \\ 4 & 2 & 1 & 22 \end{bmatrix} \quad \text{rref} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$y=3x^2+4x+2$$