

Key

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find an exact value. Use the sum or difference formulas

$$1) \sin 75^\circ \quad \sin(30+45) = \sin 30 \cdot \cos 45 + \cos 30 \cdot \sin 45 \rightarrow$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$1) \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$2) \cos 15^\circ$$

$$\cos(45-30) = \cos 45 \cdot \cos 30 + \sin 45 \cdot \sin 30 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$2) \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$3) \cos \frac{\pi}{12} \quad \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cdot \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \cdot \sin \frac{\pi}{4}$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$3) \frac{\sqrt{2} + \sqrt{6}}{4}$$

Write the expression as the sine, cosine, or tangent of an angle.

$$4) \sin 61^\circ \cos 19^\circ - \cos 61^\circ \sin 19^\circ \quad \sin(61-19)$$

$$4) \sin 42^\circ$$

$$5) \sin \frac{\pi}{3} \cos \frac{\pi}{5} + \cos \frac{\pi}{3} \sin \frac{\pi}{5} \quad \sin\left(\frac{\pi}{3} + \frac{\pi}{5}\right)$$

$$\frac{5\pi}{15} + \frac{3\pi}{15} = \frac{8\pi}{15}$$

$$5) \sin \frac{8\pi}{15}$$

Prove the identity.

$$6) \cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$6) \underline{\hspace{2cm}}$$

$$\cos x \cdot \cos \frac{\pi}{2} - \sin x \cdot \sin \frac{\pi}{2}$$

$$\cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x \quad \square$$

$$7) \cos(x-y) - \cos(x+y) = 2 \sin x \sin y$$

$$7) \underline{\hspace{2cm}}$$

$$\cos x \cdot \cos y + \sin x \cdot \sin y - (\cos x \cdot \cos y - \sin x \cdot \sin y)$$

$$\cos x \cdot \cos y + \sin x \cdot \sin y - \cos x \cdot \cos y + \sin x \cdot \sin y = 2 \sin x \cdot \sin y \quad \square$$

Rewrite with only sin x and cos x.

$$8) \sin 2x - \cos 2x \quad 2 \sin x \cdot \cos x - \cos^2 x + \sin^2 x$$

$$8) \underline{\hspace{2cm}}$$

Find the exact value by using a half-angle identity.

$$9) \sin \frac{\pi}{12} \quad \sin \frac{\pi}{12} = \pm \sqrt{\frac{1 - \cos \frac{\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{2}}$$

$$9) \frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$10) \cos\left(-\frac{\pi}{8}\right)$$

$$10) \underline{\hspace{2cm}}$$

$$\cos\left(-\frac{\pi}{8}\right) = \pm \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2}{2} + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{\sqrt{2}}$$

$$\frac{\sqrt{2 + \sqrt{2}}}{2}$$

Prove the identity.

11) $\sin 4u = 2 \sin 2u \cos 2u$

11) _____

$$\begin{aligned}\sin(4u) &= \sin(2u+2u) \text{ using sin sum} \\ \sin 2u \cdot \cos 2u + \cos 2u \cdot \sin 2u \\ &= 2 \sin 2u \cos 2u \quad \square\end{aligned}$$

Find all solutions to the equation in the interval $[0, 2\pi)$.

12) $\sin 2x = -\sin x$

12) _____

$$2 \sin x \cos x + \sin x = 0 \quad \frac{2 \cos x}{2} = -\frac{1}{2}$$

$$\sin x (2 \cos x + 1) = 0 \quad \cos x = -\frac{1}{2}$$

$$\sin x = 0 \quad \boxed{x = 0, \pi}$$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

13) $2 \cos x + \sin 2x = 0$

13) _____

$$2 \cos x + 2 \sin x \cdot \cos x = 0$$

$$2 \cos x (1 + \sin x) = 0$$

$$\cos x = 0$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$