

Precalculus Review 5.1-5.2

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 1) List the 3 pythagorean identities.

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \\ \cot^2 x + 1 &= \csc^2 x\end{aligned}$$

Use the fundamental identities to find the value of the trigonometric function.

- 2) Given that $\sin \theta = \frac{1}{5}$, find $\sec(\pi/2 - \theta)$.

$$\boxed{5}$$

Use basic identities to simplify the expression.

- 3) $\cot \theta \sec \theta \sin \theta$

$$\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} = \boxed{1}$$

Find all solutions in the interval $[0, 2\pi)$.

- 4) $\cos^2 x + 2 \cos x + 1 = 0$

$$\begin{aligned}(\cos x + 1)(\cos x + 1) &= 0 \\ \cos x + 1 &= 0 \\ \cos x &= -1\end{aligned}$$

$$\boxed{x = \pi}$$

- 5) $2 \sin^2 x = \sin x$

$$\begin{aligned}2\sin^2 x - \sin x &= 0 \\ \sin x(2\sin x - 1) &= 0 \\ \sin x &= 0 \quad 2\sin x - 1 = 0 \\ \sin x &= 0 \quad \sin x = \frac{1}{2}\end{aligned}$$

$$\boxed{x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}}$$

- 6) $4 \sin^2 x - 4 \sin x + 1 = 0$

$$(2\sin x - 1)(2\sin x - 1) = 0$$

Use basic identities to simplify the expression.

- 7) $\frac{\tan \theta}{\sec \theta}$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} = \boxed{\sin \theta}$$

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

Simplify the expression. Use Pythagorean Identities.

- 8) $(\sin^2 x + \cos^2 x) - (\csc^2 x - \cot^2 x)$

$$1 - \csc^2 x + \cot^2 x$$

$$1 - (\cot^2 x + 1) + \cot^2 x$$

$$1 - \cot^2 x - 1 + \cot^2 x$$

$$1 - 1 - \cot^2 x + \cot^2 x$$

$$\boxed{0}$$

Prove the identity.

9) $\sin x \sec x \cot x = 1$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = 1 \quad \checkmark$$

Determine if the following is an identity.

10) $\tan^2 x = \sec^2 x - \sin^2 x - \cos^2 x$

Prove the identity.

$$\begin{aligned} 11) \frac{\cot x}{1 + \csc x} &= \frac{\csc x - 1}{\cot x} \left(\frac{\csc x + 1}{\csc x + 1} \right) \\ &= \frac{\csc^2 x - 1}{\cot x (\csc x + 1)} \\ &= \frac{\cot^2 x}{\cot x (\csc x + 1)} \\ &= \frac{\cot x}{\csc x + 1} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \tan^2 x &= (1 + \tan^2 x) - \sin^2 x - \cos^2 x \\ &= (1 + \tan^2 x) - (1 - \cos^2 x) - \cos^2 x \\ &= 1 + \tan^2 x - 1 + \cos^2 x - \cos^2 x \\ &= \tan^2 x \quad \checkmark \end{aligned}$$