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12. Given: $C = 103°, b = 46, c = 61$ — an SSA case. $h = b \sin C \approx 44.8$; $h < b < c$, so there is one triangle.

$$B = \sin^{-1}\left(\frac{b \sin C}{c}\right) = \sin^{-1}(0.734\ldots) \approx 47.3°$$
$$A = 180° - (B + C) = 29.7°;$$
$$a = \frac{c \sin A}{\sin C} = \frac{61 \sin 29.7°}{\sin 103°} \approx 31.0$$

13. Given: $A = 36°, a = 2, b = 7$. $h = b \sin A \approx 4.1$; $a < h$, so no triangle is formed.

14. Given: $B = 82°, b = 17, c = 15$. $h = c \sin B \approx 14.9$; $h < c < b$, so there is one triangle.

15. Given: $C = 36°, a = 17, c = 16$. $h = a \sin C \approx 10.0$; $h < c < a$, so there are two triangles.

16. Given: $A = 73°, a = 24, b = 28$. $h = b \sin A \approx 26.8$; $a < h$, so no triangle is formed.

17. Given: $C = 30°, a = 18, c = 9$. $h = a \sin C \approx 9$; $h = c$, so there is one triangle.

18. Given: $B = 88°, b = 14, c = 62$. $h = c \sin B \approx 62.0$; $b < h$, so no triangle is formed.

19. Given: $A = 64°, a = 16, b = 17$. $h = b \sin A \approx 15.3$; $h < a < b$, so there are two triangles.

$$B_1 = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.954\ldots) \approx 72.7°$$
$$C_1 = 180° - (A + B_1) \approx 43.3°;$$
$$c_1 = \frac{a \sin C_1}{\sin A} = \frac{16 \sin 43.3°}{\sin 64°} \approx 12.2$$

Or (with B obtuse):

$$B_2 = 180° - B_1 \approx 107.3°;$$
$$C_2 = 180° - (A + B_2) \approx 8.7°;$$
$$c_2 = \frac{a \sin C_2}{\sin A} \approx 2.7$$

20. Given: $B = 38°, b = 21, c = 25$. $h = c \sin B \approx 15.4$; $h < b < c$, so there are two triangles.

$$C_1 = \sin^{-1}\left(\frac{c \sin B}{b}\right) = \sin^{-1}(0.732\ldots) \approx 47.1°$$
$$A_1 = 180° - (B + C_1) \approx 94.9°;$$
$$a_1 = \frac{b \sin A_1}{\sin B} = \frac{21 \sin 94.9°}{\sin 38°} \approx 34.0$$

Or (with C obtuse):

$$C_2 = 180° - C_1 \approx 132.9°;$$
$$A_2 = 180° - (B + C_2) \approx 9.1°;$$
$$a_2 = \frac{b \sin A_2}{\sin B} \approx 5.4$$

21. Given: $C = 68°, a = 19, c = 18$. $h = a \sin C \approx 17.6$; $h < c < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin C}{c}\right) = \sin^{-1}(0.978\ldots) \approx 78.2°$$
$$B_1 = 180° - (A + C) \approx 33.8°;$$
$$b_1 = \frac{c \sin B_1}{\sin C} = \frac{18 \sin 33.8°}{\sin 68°} \approx 10.8$$

Or (with A obtuse):

$$A_2 = 180° - A_1 \approx 101.8°;$$
$$B_2 = 180° - (A_2 + C) \approx 10.2°;$$
$$b_2 = \frac{c \sin B_2}{\sin C} \approx 3.4$$

22. Given: $B = 57°, a = 11, b = 10$. $h = a \sin B \approx 9.2$; $h < b < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.922\ldots) \approx 67.3°$$
$$C_1 = 180° - (A_1 + B) \approx 55.7°;$$
$$c_1 = \frac{b \sin C_1}{\sin B} = \frac{10 \sin 55.7°}{\sin 57°} \approx 9.9$$

Or (with A obtuse):

$$A_2 = 180° - A_1 \approx 112.7°;$$
$$C_2 = 180° - (A_2 + B) \approx 10.3°;$$
$$c_2 = \frac{b \sin C_2}{\sin B} \approx 2.1$$

23. $h = 10 \sin 42° \approx 6.691$, so:

(a) $6.691 < b < 10$.

(b) $b \approx 6.691$ or $b \geq 10$.

(c) $b < 6.691$

24. $h = 12 \sin 53° \approx 9.584$, so:

(a) $9.584 < c < 12$.

(b) $c \approx 9.584$ or $c \geq 12$.

(c) $c < 9.584$

25. **(a)** No: this is an SAS case

(b) No: only two pieces of information given.

26. **(a)** Yes: this is an AAS case.

$$B = 180° - (A + C) = 32°;$$
$$b = \frac{a \sin B}{\sin A} = \frac{81° \sin 32°}{\sin 29°} \approx 88.5;$$
$$c = \frac{a \sin C}{\sin A} = \frac{81 \sin 119°}{\sin 29°} \approx 146.1$$

(b) No: this is an SAS case.

27. Given: $A = 61°, a = 8, b = 21$ — an SSA case. $h = b \sin A = 18.4$; $a < h$, so no triangle is formed.

28. Given: $B = 47°, a = 8, b = 21$ — an SSA case. $h = a \sin B \approx 5.9$; $h < a < b$, so there is one triangle.

$$A = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.278\ldots) \approx 16.2°$$
$$C = 180° - (A + B) = 116.8°;$$
$$c = \frac{b \sin C}{\sin B} = \frac{21 \sin 116.8°}{\sin 47°} \approx 25.6$$

29. Given: $A = 136°, a = 15, b = 28$ — an SSA case. $h = b \sin A \approx 19.5$; $a < h$, so no triangle is formed.

30. Given: $C = 115°, b = 12, c = 7$ — an SSA case. $h = b \sin C \approx 10.9$; $c < h$, so no triangle is formed.

31. Given: $B = 42°, c = 18, C = 39°$ — an AAS case.

$$A = 180° - (B + C) = 99°;$$
$$a = \frac{c \sin A}{\sin C} = \frac{18 \sin 99°}{\sin 39°} \approx 28.3;$$
$$b = \frac{c \sin B}{\sin C} = \frac{18 \sin 42°}{\sin 39°} \approx 19.1$$

32. Given: $A = 19°, b = 22, B = 47°$ — an AAS case.

$$C = 180° - (A + B) = 114°;$$
$$a = \frac{b \sin A}{\sin B} = \frac{22 \sin 19°}{\sin 47°} \approx 9.8;$$
$$c = \frac{b \sin C}{\sin B} = \frac{22 \sin 114°}{\sin 47°} \approx 27.5$$

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41. The left side factors to $(\sin x - 3)(\sin x + 1) = 0$; only $\sin x = -1$ is possible, so $x = \frac{3\pi}{2}$.

42. $2\cos^2 t - 1 = \cos t$, or $2\cos^2 t - \cos t - 1 = 0$, or $(2\cos t + 1)(\cos t - 1) = 0$. Then $\cos t = -\frac{1}{2}$ or $\cos t = 1$: $t = 0, t = \frac{2\pi}{3}$ or $t = \frac{4\pi}{3}$.

43. $\sin(\cos x) = 1$ only if $\cos x = \frac{\pi}{2} + 2n\pi$. No choice of n gives a value in $[-1, 1]$, so there are no solutions.

44. $\cos(2x) + 5\cos x - 2 = 2\cos^2 x - 1 + 5\cos x - 2$ $= 2\cos^2 x + 5\cos x - 3 = 0$. $(2\cos x - 1)(\cos x + 3)$ $= 0$, so $\cos(x) = \frac{1}{2}$ and $\cos(x) = -3$. The latter is extraneous so $x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

For #45–48, use graphs to suggest the intervals. To find the endpoints of the intervals, treat the inequalities as equations and solve.

45. $\cos 2x = \frac{1}{2}$ has solutions $x = \frac{\pi}{6}, x = \frac{5\pi}{6}, x = \frac{7\pi}{6}$, and $x = \frac{11\pi}{6}$ in interval $[0, 2\pi)$. The solution set for the inequality is, $0 \le x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x < \frac{7\pi}{6}$ or $\frac{11\pi}{6} < x < 2\pi$; that is, $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$.

46. $2\sin x \cos x = 2\cos x$ is equivalent to $(\cos x)(\sin x - 1) = 0$, so the solutions in $(0, 2\pi]$ are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. The solution set for the inequality is $\frac{\pi}{2} < x < \frac{3\pi}{2}$; that is, $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

47. $\cos x = \frac{1}{2}$ has solutions $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ in the interval $[0, 2\pi)$. The solution set for the inequality is $\frac{\pi}{3} < x < \frac{5\pi}{3}$; that is, $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$.

48. $\tan x = \sin x$ is equivalent to $(\sin x)(\cos x - 1) = 0$, so the only solution in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is $x = 0$. The solution set for the inequality is $-\frac{\pi}{2} < x < 0$; that is, $\left(-\frac{\pi}{2}, 0\right)$.

49. $y = 5\sin(3x + \cos^{-1}(3/5)) \approx 5\sin(3x + 0.9273)$

50. $y = 13\sin(2x - \cos^{-1}(5/13)) \approx 13\sin(2x - 1.176)$

51. Given: $A = 79°, B = 33°, a = 7$ — an AAS case.
$C = 180° - (A + B) = 68°$
$b = \frac{a \sin B}{\sin A} = \frac{7 \sin 33°}{\sin 79°} \approx 3.9$;
$c = \frac{a \sin C}{\sin A} = \frac{7 \sin 68°}{\sin 79°} \approx 6.6$.

52. Given: $a = 5, b = 8, B = 110°$ — an SSA case. Using the law of sines: $h = a \sin B = 4.7$; $h < a < b$, so there is one triangle.
$A = \sin^{-1}\left(\frac{a \sin B}{b}\right) \approx \sin^{-1}(0.587) \approx 36.0°$;
$C = 180° - (A + B) \approx 34.0°$;
$c = \frac{b \sin C}{\sin B} \approx \frac{8 \sin 34.0°}{\sin 110°} \approx 4.8$.
Using law of cosines: Solve the quadratic equation $8^2 = 5^2 + c^2 - 2(5)c\cos 110°$, or $c^2 + (3.420)c - 39 = 0$; there is one positive solution: $c \approx 4.8$. Since $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$:
$A \approx \cos^{-1}(0.809) \approx 36.0°$ and $C = 180° - (A + B)$ $\approx 34.0°$.

53. Given: $a = 8, b = 3, B = 30°$ — an SSA case. Using the law of sines: $h = a \sin B = 4$; $b < h$, so no triangle is formed. Using the law of cosines: Solve the quadratic equation $3^2 = 8^2 + c^2 - 2(8)c\cos 30°$, or $c^2 - (8\sqrt{3})c + 55 = 0$; there are no real solutions.

54. Given: $a = 14.7, A = 29.3°, C = 33°$ — an AAS case.
$B = 180° - (A + C) = 117.7°$, and
$b = \frac{a \sin B}{\sin A} = \frac{14.7 \sin 117.7°}{\sin 29.3°} \approx 26.6$;
$c = \frac{a \sin C}{\sin A} = \frac{14.7 \sin 33°}{\sin 29.3°} \approx 16.4$.

55. Given: $A = 34°, B = 74°, c = 5$ — an ASA case.
$C = 180° - (A + B) = 72°$;
$a = \frac{c \sin A}{\sin C} = \frac{5 \sin 34°}{\sin 72°} \approx 2.9$;
$b = \frac{c \sin B}{\sin C} = \frac{5 \sin 74°}{\sin 72°} \approx 5.1$.

56. Given: $c = 41, A = 22.9°, C = 55.1°$ — an AAS case.
$B = 180° - (A + C) = 102°$;
$a = \frac{c \sin A}{\sin C} = \frac{41 \sin 22.9°}{\sin 55.1°} \approx 19.5$;
$b = \frac{c \sin B}{\sin C} = \frac{41 \sin 102°}{\sin 55.1°} \approx 48.9$.

57. Given: $a = 5, b = 7, c = 6$ — an SSS case.
$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.714) \approx 44.4°$;
$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.2) \approx 78.5°$;
$C = 180° - (A + B) \approx 57.1°$.

58. Given: $A = 85°, a = 6, b = 4$ — an SSA case. Using the law of sines: $h = b \sin A \approx 4.0$; $h < b < a$, so there is one triangle.
$B = \sin^{-1}\left(\frac{b \sin A}{a}\right) \approx \sin^{-1}(0.664) \approx 41.6°$;
$C = 180° - (A + B) \approx 53.4°$;
$c = \frac{a \sin C}{\sin A} = \frac{6 \sin 53.4°}{\sin 85°} \approx 4.8$.

Using the law of cosines: Solve the quadratic equation $6^2 = 4^2 + c^2 - 2(4)c\cos 85°$, or $c^2 - (0.697)c - 20 = 0$; there is one positive solution:
$c \approx 4.8$. Since $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$:
$B \approx \cos^{-1}(0.747) \approx 41.6°$ and $C = 180° - (A + B)$
$\approx 53.4°$.

59. $s = \frac{1}{2}(3 + 5 + 6) = 7$;
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{7(7-3)(7-5)(7-6)} = \sqrt{56} \approx 7.5$$

60. $c \approx 7.672$ so Area $\approx \sqrt{528.141} \approx 23.0$ (using Heron's formula). Or, use $A = \frac{1}{2}ab\sin C$.

61. $h = 12\sin 28° \approx 5.6$, so:
(a) $\approx 5.6 < b < 12$.
(b) $b \approx 5.6$ or $b \geq 12$.
(c) $b < 5.6$.

62. **(a)** $C = 180° - (A + B) = 45°$, so
$$AC = b = \frac{c\sin B}{\sin C} = \frac{80\sin 65°}{\sin 45°} \approx 102.5 \text{ ft.}$$
(b) The distance across the canyon is $b\sin A \approx 96.4$ ft.

63. Given: $c = 1.75$, $A = 33°$, $B = 37°$ — an ASA case, so
$C = 180° - (A + B) = 110°$;
$$a = \frac{c\sin A}{\sin C} = \frac{1.75\sin 33°}{\sin 110°} \approx 1.0;$$
$$b = \frac{c\sin B}{\sin C} = \frac{1.75\sin 37°}{\sin 110°} \approx 1.1,$$
and finally, the height is $h = b\sin A = a\sin b \approx 0.6$ mi.

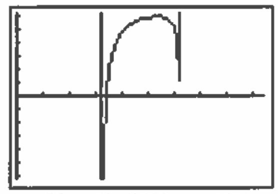
64. Given: $C = 70°$, $a = 225$, $b = 900$ — an SAS case, so
$AB = c = \sqrt{a^2 + b^2 - 2ab\cos C}$
$\approx \sqrt{722{,}106.841} \approx 849.77$ ft.

65. Let $a = 8$, $b = 9$, and $c = 10$. The largest angle is opposite the largest side, so we call it C.
Since $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$, $C = \cos^{-1}\left(\dfrac{5}{16}\right)$
$\approx 71.790°$, 1.253 rad.

66. The shorter diagonal splits the parallelogram into two (congruent) triangles with $a = 15$, $B = 40°$, and $c = 24$. The shorter diagonal has length
$b = \sqrt{a^2 + b^2 - 2ac\cos B} \approx \sqrt{249.448} \approx 15.794$ ft.
Since adjacent angles are supplementary, the other angle is 140°. The longer diagonal splits the parallelogram into (two) congruent triangles with $a = 15$, $B = 140°$, and $c = 24$, so the longer diagonal length is
$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{1352.552} \approx 36.777$ ft.

67. **(a)** The point (x, y) has coordinates $(\cos\theta, \sin\theta)$, so the bottom is $b_1 = 2$ units wide, the top is $b_2 = 2x = 2\cos\theta$ units wide, and the height is $h = y = \sin\theta$ units. Either use the formula for the area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, or notice that the trapezoid can be split into two triangles and a rectangle. Either way:
$$A(\theta) = \sin\theta + \sin\theta\cos\theta = \sin\theta(1 + \cos\theta) = \sin\theta + \frac{1}{2}\sin 2\theta.$$
(b) The maximizing angle is $\theta = \frac{\pi}{3} = 60°$; the maximum area is $\frac{3}{4}\sqrt{3} \approx 1.30$ square units.

68. **(a)** Substituting the values of a and b:
$$S(\theta) = 6.825 + 0.63375(-\cot\theta + \sqrt{3}\cdot\csc\theta) = 6.825 + \frac{0.63375(\sqrt{3} - \cos\theta)}{\sin\theta}$$



[0, 9.4] by [−6.2, 6.2]

(b) Considering only angles between 0 and π, the minimum occurs when $\theta \approx 0.96 \approx 54.74°$.
(c) The minimum value of S is approximately $S(0.96) \approx 7.72$ in.^{2}

69. **(a)** Split the quadrilateral in half to leave two (identical) right triangles, with one leg 4000 mi, hypotenuse $4000 + h$ mi, and one acute angle $\theta/2$.
Then $\cos\frac{\theta}{2} = \dfrac{4000}{4000 + h}$; solve for h to leave
$$h = \frac{4000}{\cos(\theta/2)} - 4000 = 4000\sec\frac{\theta}{2} - 4000 \text{ miles.}$$
(b) $\cos\frac{\theta}{2} = \frac{4000}{4200}$, so $\theta = 2\cos^{-1}\left(\frac{20}{21}\right) \approx 0.62 \approx 35.51°$.

70. Using the double angle sine formula, we rewrite the left side:
$\sin x - 2\sin x\cos x + 3\cos^2 x\sin x - \sin^3 x$
$= (\sin x)(1 - 2\cos x + 3\cos^2 x - \sin^2 x)$
$= (\sin x)(4\cos^2 x - 2\cos x)$
$= (2\sin x\cos x)(2\cos x - 1) = (\sin 2x)(2\cos x - 1)$.
This equals 0 when $x = n\frac{\pi}{2}$ or $x = \pm\frac{\pi}{3} + 2n\pi$, n an integer.

71. The hexagon is made up of 6 equilateral triangles; using Heron's formula (or some other method), we find that each triangle has area $\sqrt{24(24 - 16)^3}$ $= \sqrt{12{,}288} = 64\sqrt{3}$. The hexagon's area is therefore, $384\sqrt{3}$ cm^2, and the radius of the circle is 16 cm, so the area of the circle is 256π cm^2, and the area outside the hexagon is $256\pi - 384\sqrt{3} \approx 139.140$ cm^2.

72. The pentagon is made up of 5 triangles with base length 12 cm and height $\dfrac{6}{\tan 36°} \approx 8.258$ cm, so its area is about 247.749 cm^2. The radius of the circle is the height of those triangles, so the desired area is about $247.749 - \pi(8.258)^2$ ≈ 33.494 cm^2.